Homework #0

1. (a) Find an equation for the line $L(x)$ that interpolates \[
\begin{array}{c|cc}
x & a & b \\
y & A & B \\
\end{array}
\]
(b) Given $f(x) = \sin x$, find the line $L(x)$ such that the range of $L \circ f$ is $[-2, 4]$. Also find the value of $L(f(1))$.

2. (a) Let $f(x) = \sin x$ and consider evenly-spaced nodes $0 = x_0 < x_1 < x_2 < x_3 < x_4 = \pi$. Find equations for the piecewise linear interpolant $P(x)$ for the data $(x_i, f(x_i))$, for $i = 0, \ldots, 4$.

(b) Consider evenly-spaced nodes $0 = z_0 < z_1 < z_2 < z_3 = \pi$. Find $P(z_i)$, for $i = 0, \ldots, 3$, and find the absolute errors $|f(z_i) - P(z_i)|$, for $i = 0, \ldots, 3$.

3. Let $f(x) = \sin x$ and consider evenly-spaced nodes $0 = x_0 < x_1 < x_2 < x_3 < x_4 = 2\pi$. Let $y_i = f(x_i)$, for $i = 0, \ldots, 4$. Use the equation

$$z_i = \frac{y_{i-1} + y_{i+1}}{2}$$

to find $z_i$, for $i = 1, \ldots, 3$.

4. Show, using integration by parts, that

$$\int_a^b f'(x)g(x) \, dx = -\int_a^b f(x)g'(x) \, dx + f(b)g(b) - f(a)g(a).$$

5. (a) Consider an evenly-spaced grid in $[0, 2\pi] \times [0, 2\pi]$, consisting of nodes $(x_i, y_j)$, where $x_i = ih$ and $y_j = jh$ and $h = 2\pi/99$, for $i, j = 0, \ldots, 99$. Make a square $100 \times 100$ image over this grid with intensities $|\sin x \sin y|$.

(b) Similarly, make a rectangular $400 \times 200$ image (width of 400 and height of 200) in $[-2, 2] \times [-1, 1]$ with intensities $|\sin x \sin y|$.

6. Make a square $250 \times 250$ image, $f$, with a white background and a black inscribed circle.

7. Download a color .bmp image. Can our program read in such an image? What does the output look like? Now, double the intensities and look at the image. Instead, halve the intensities and look at the image.