Homework #3

1. Resize, using bicubic interpolation, and von Neumann boundary conditions, the following images according to the following instructions. For each, submit the image and write down its maximum and minimum intensity values.

(a) $211 \times 211$ image in $[−2, 2] \times [−2, 2]$, with intensity $0.3$ for $x \leq 0$, and $0.7$ otherwise, into a $433 \times 433$ image.

(b) $211 \times 211$ image in $[−1, 1] \times [−1, 1]$, with a circle of radius 1 about the origin of intensity $0.25$, on a background of intensity $0.75$, into a $433 \times 613$ image.

(c) $211 \times 211$ image in $[−1, 1] \times [−1, 1]$, with intensity $|\sin(\pi x)\sin(\pi y)|$, into a $613 \times 433$ image.

2. Resize, using bicubic interpolation, and von Neumann boundary conditions, the following images according to the following instructions. For each, submit the image and write down the intensity at pixel $(17, 17)$.

(a) drapes.bmp, into an image of width 600 and height 400.

(b) plant.bmp, into an image of width 800 and height 400.

(c) octopus.bmp, into an image swapping the width and height.

(d) cat.bmp, into an image with half the width and half the height (rounded up).

3. Let $f : [−1, 1] \times [−1, 1] \rightarrow [0, 1]$ and suppose the image is corrupted in the domain $D \subseteq [−1, 1] \times [−1, 1]$. Consider inpainting by replacing intensity values in $D$ with those from the steady state heat equation:

$$f_{xx} + f_{yy} = 0,$$

where we use von Neumann boundary conditions at the boundary of $[−1, 1] \times [−1, 1]$. Now consider the following procedure:

- Replace values of $f$ in $D$ by 0.5.
- Form the linear system of equations that has equation, at pixel $(i, j)$,

$$\frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h^2} + \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j+1}}{h^2} = 0,$$

using a coordinate list format on the sparse matrix.
- Use Jacobi iterations on the linear system (for initial guess, use the corrupted intensities 0.5), until

$$||f^{(n)} - f^{(n-1)}||_\infty < 10^{-4}.$$ 

Apply the procedure to the following images, with the following corrupted domains $D$. Submit your final image and write down the number of iterations and the final infinity norm error.
(a) $500 \times 500$ image with intensities 0.25, for $x \leq 0$; 0.75 otherwise, and $D = \{(x, y)| -\frac{1}{3} \leq x \leq \frac{1}{3}, -\frac{1}{3} \leq y \leq \frac{1}{3}\}$.

(b) $500 \times 500$ image with intensities 0.75, for $x \leq -\frac{1}{3}$; 0.25 for $-\frac{1}{3} < x \leq \frac{1}{3}$; 0.5 otherwise, and $D = \{(x, y)| -\frac{1}{3} \leq y \leq \frac{1}{3}\}$.

(c) $500 \times 500$ image with intensities 0.75, for $x \leq -\frac{1}{3}$; 0.25 for $-\frac{1}{3} < x \leq \frac{1}{3}$; 0.5 otherwise, and where $D$ includes all pixels $(i, j)$ satisfying $j$ or $j+1$ divisible by 5.

4. Use the same procedure as in the previous problem on the following images $f : R \to [0,1]$, with the following corrupted domains $D$. Submit your final image and write down the number of iterations and the final infinity norm error.

(a) drapes.bmp, where $D$ includes all pixels $(i, j)$ satisfying $j$ divisible by 2.

(b) plant.bmp, where $D$ includes all pixels $(i, j)$ satisfying $i$ or $i+1$ or $i+2$ divisible by 7, or $j$ divisible by 4.

(c) octopus.bmp, where $D$ includes all pixels $(i, j)$ satisfying $i \leq 50$ and $j \leq 50$. 