Homework #3

Turn in the problems marked with a (*).

1. Show the number of flops needed to perform Gaussian elimination without pivoting on an $n \times n$ banded matrix A, with upper and lower bandwidth r, is $bigO(nr^2)$.

2. (*) Let A be an $n \times n$ sparse matrix with m_i number of nonzeros in the ith row, including one on the diagonal. Count the number of flops, in terms of the total number of nonzero elements in A, needed to perform one step of Jacobi iterative method.

3. Form the $n \times n$ tridiagonal matrix, T, with 0 on the main diagonal, and 1/2 on the upper and lower diagonal. Let A be the $n \times n$ tridiagonal matrix with -2 on the diagonal and 1 on the upper and lower diagonal.

   (a) What role does this matrix play in a Jacobi iterative method for a linear system $Ax = b$?

   (b) Use Matlab to find the 2-norm of the matrix when $n = 100, 200, 400$.

   (c) For the result at $n = 100$, estimate, using the bound

   $$||x^{(k)} - x||_2 \leq ||T||_2^k ||x^{(0)} - x||_2$$

   how many iterations k are needed to force $||x^{(k)} - x||_2 < 10^{-5}$, when $||x^{(0)} - x||_2 = 1$.

4. Try the latter two parts of problem #3, but using the Gauss-Seidel iterative method.

5. (*) Try the latter two parts of problem #3, but using the SOR iterative method with $\omega = 1.5$.

6. Try the latter two parts of problem #3, but only for the 100×100 block tridiagonal matrix A with 10×10 blocks of B on the main diagonal and identity matrix on the upper and lower diagonals, where B is tridiagonal with -4 on its main diagonal, and 1 on its upper and lower diagonals.