Homework #3

Turn in the problems marked with a (*).

1. Show the number of flops needed to perform Gaussian elimination without pivoting on an \( n \times n \) banded matrix \( A \), with upper and lower bandwidth \( r \), is \( \text{bigO}(nr^2) \).

2. (*) Let \( A \) be an \( n \times n \) sparse matrix with \( m_i \) number of nonzeros in the \( i \)th row, including one on the diagonal. Count the number of flops, in terms of the total number of nonzero elements in \( A \), needed to perform one step of Jacobi iterative method.

3. Form the \( n \times n \) tridiagonal matrix, \( T \), with 0 on the main diagonal, and \( 1/2 \) on the upper and lower diagonal. Let \( A \) be the \( n \times n \) tridiagonal matrix with \( -2 \) on the diagonal and \( 1 \) on the upper and lower diagonal.
   
   (a) What role does this matrix play in a Jacobi iterative method for a linear system \( Ax = b \)?
   
   (b) Use Matlab to find the 2-norm of the matrix when \( n = 100, 200, 400 \).
   
   (c) For the result at \( n = 100 \), estimate, using the bound
   
   \[
   \| x^{(k)} - x \|_2 \leq \| T \|_2^k \| x^{(0)} - x \|_2
   \]
   
   how many iterations \( k \) are needed to force \( \| x^{(k)} - x \|_2 < 10^{-5} \), when \( \| x^{(0)} - x \|_2 = 1 \).

4. Try the latter two parts of problem #3, but using the Gauss-Seidel iterative method.

5. (*) Try the latter two parts of problem #3, but using the SOR iterative method with \( \omega = 1.5 \).

6. Try the latter two parts of problem #3, but only for the \( 100 \times 100 \) block tridiagonal matrix \( A \) with \( 10 \times 10 \) blocks of \( B \) on the main diagonal and identity matrix on the upper and lower diagonals, where \( B \) is tridiagonal with \( -4 \) on its main diagonal, and \( 1 \) on its upper and lower diagonals.