Project #3

1. Suppose the computational domain D is $[-1,1] \times [-1,1]$. Choose an ellipse contained in this computational domain as interface, and denote by Ω the inside of the ellipse. We consider solving the steady state heat equation

$$\begin{cases} \Delta u = f, & u \in \Omega \\ u = 2, & u \in \partial\Omega, \end{cases}$$

and

$$\begin{cases} \Delta u = g, & u \in \Omega^c \cap D \\ u = 1, & u \in \partial\Omega \\ u = 0, & u \in \partial D. \end{cases}$$

where $f(x,y) = 4e^y \sin \pi x$ and $g(x,y) = e^x \cos 2\pi y$. Use the ghost fluid method and conjugate gradient that is preconditioned with incomplete Cholesky factorization (with no fill-ins), stopping when the relative residual ($||b - Ax||_2/||b||_2$) is $\leq 10^{-7}$, to solve for u on the grid with nodes (x_i, y_j), where

$$-1 = x_0 < x_1 < \ldots < x_{100} = 1$$
$$-1 = y_0 < y_1 < \ldots < y_{100} = 1$$

are equally spaced. Turn in the plot of u.