Project #3

1. Suppose the computational domain $D$ is $[-1, 1] \times [-1, 1]$. Choose an ellipse contained in this computational domain as interface, and denote by $\Omega$ the inside of the ellipse. We consider solving the steady state heat equation

$$\begin{cases}
\Delta u = f, & u \in \Omega \\
u = 2, & u \in \partial \Omega,
\end{cases}$$

and

$$\begin{cases}
\Delta u = g, & u \in \Omega^c \cap D \\
u = 1, & u \in \partial \Omega \\
u = 0, & u \in \partial D.
\end{cases}$$

where $f(x, y) = 4e^y \sin \pi x$ and $g(x, y) = e^x \cos 2\pi y$. Use the ghost fluid method and conjugate gradient that is preconditioned with incomplete Cholesky factorization (with no fill-ins), stopping when the relative residual ($||b - Ax||_2/||b||_2$) is $\leq 10^{-7}$, to solve for $u$ on the grid with nodes $(x_i, y_j)$, where

$$-1 = x_0 < x_1 < \ldots < x_{100} = 1$$
$$-1 = y_0 < y_1 < \ldots < y_{100} = 1$$

are equally spaced. Turn in the plot of $u$. 