1. Expand and simplify the curvature vector
\[
\frac{1}{|\vec{x}|} \left( \frac{(\vec{x}')^\perp}{|\vec{x}|} \right)'.
\]

2. Consider velocity field \( v(\vec{x}) = [\cos x_2, 1]^T \), and let a point initially at \( \vec{z} \) flow along this velocity field for time \( t \). Find the analytic form for the final position of the point, using the ODE
\[
\vec{x}_t = v(\vec{x}).
\]

3. Consider the initial parametrized curve \([\cos \theta, \sin \theta]^T\), for \( \theta \in [0, 2\pi] \), and discretize it using evenly spaced
\[
0 = \theta_0 < \ldots < \theta_{100} = 2\pi.
\]
Program up front tracking for transport equation under velocity field \( v(\vec{x}) = [\cos x_2, 1]^T \), using Euler’s method as ODE solver. Apply it to the discretized curve up to time 1, using time steps of \( \Delta t \).

(a) Plot your result for \( \Delta t = 0.04 \).

(b) For each of \( \Delta t = 0.04, 0.02, 0.01 \), calculate the absolute errors (using the analytic solution) for each point of the discretized curve at the final time and write down the maximum, among all the points, of these absolute errors.

4. Do the same for front tracking for transport equation using the Runge-Kutta order 2 midpoint method as ODE solver.