Homework 2/1/19

1. Consider the graph of a function $x_2 = f(x_1)$, represented by parametrization $x_1(\theta) = \theta, x_2(\theta) = f(\theta)$, and by level-set function $\phi(x_1, x_2) = x_2 - f(x_1)$. Given $\theta$, show

$$-\left(\nabla \cdot \frac{\nabla \phi}{|\nabla \phi|}\right) \frac{\nabla \phi}{|\nabla \phi|},$$

evaluated at $\bar{x}(\theta)$, equals to the curvature vector

$$\frac{1}{|\bar{x}'(\theta)|} \begin{pmatrix} \bar{x}'(\theta) \\ |\bar{x}(\theta)| \end{pmatrix}.$$

2. Use first order upwind differencing in space, and Euler’s method as ODE solver, to approximate the transport equation

$$\phi_t + \vec{v} \cdot \nabla \phi = 0.$$

Apply it to a circle of radius 0.3 for $\vec{v}(\bar{x}) = \bar{x}$ in the domain $[-1, 1]^2$, using $\Delta t = h^2/\max |\vec{v}|$, up to time $t = 0.7$, for $h = 0.01$.

(a) Plot the final curve.

(b) What is the exact solution?

(c) For each gridline, find the roots of the piecewise linear approximation. What is the maximum absolute error of all these roots?

3. Repeat the previous problem’s part (a) for a more complicated initial curve and velocity field.