1. Use central differencing in space, and Euler’s method as ODE solver, to approximate the curve shortening flow

\[ \phi_t = \Delta \phi_0 - \frac{\nabla \phi^T \nabla^2 \phi \nabla \phi}{|\nabla \phi|^2 + \epsilon^2}, \]

where \( \epsilon = 1.0 \times 10^{-11} \). Apply it to a circle of radius 0.5 as initial curve. Choose a stepsize \( h \) and a number of time steps to iterate, using \( \Delta t = h^2 / 4 \).

(a) Plot the final curve for your choices.

(b) Do an order of accuracy analysis by finding the error of your result, then finding the error with half the stepsize and four times as many iterations (to reach the same time).

2. Apply the flow

\[ \phi_t = \Delta \phi_0 - \frac{\nabla \phi^T \nabla^2 \phi \nabla \phi}{|\nabla \phi|^2 + \epsilon^2}, \]

in three-dimensional ambient space, starting with a dumbbell (create a dumbbell shape), and running until topological change. Plot your initial shape and final shape.