

## Math 130A Practice Midterm, Winter 2003, Lindblad.

The problems below are standard (but not easy). On the exam there might also be an additional more difficult problem for those who want the highest grade. It is recommended that you use Cramer's rule (see an appendix to the book) to calculate the inverse of the operators involved in order to save time. Note also that it is easy to come up with more similar problems yourself. Just start with something in real standard form and choose a new basis and use matlab or mathematica to calculate a similarity transformation of the matrix. Then try to go backwards without using mathematica. (This is how I came up with these problems.) To prepare for the exam play around with systems of different form and order.

1. Let  $T$  be an operator which in the standard basis has the matrix

$$A = \begin{bmatrix} 2 & 0 & -3 \\ -1 & 3 & -7 \\ 1 & 0 & 2 \end{bmatrix}$$

and the characteristic polynomial  $p(t) = (t - 3)((t - 2)^2 + 3)$ .

(a) Write  $T = S + N$ , where  $S$  is semisimple,  $N$  is nilpotent and  $SN = NS$  and give the real standard form of  $S$ .

(b) Calculate the exponential matrix  $e^{tT}$  and give the general solution of the system  $x' = Tx$ .

2. Let  $T$  be an operator which in the standard basis has the matrix

$$A = \begin{bmatrix} 1 & -4 & 2 \\ -2 & 5 & 2 \\ -12 & -12 & 15 \end{bmatrix}$$

and the characteristic polynomial  $p(t) = (t - 3)(t - 9)^2$ .

(a) Write  $T = S + N$ , where  $S$  is semisimple,  $N$  is nilpotent and  $SN = NS$  and give the real standard form of  $S$ .

(b) Calculate the exponential matrix  $e^{tT}$  and give the general solution of the system  $x' = Tx$ .