Supplementary problems for sections 3.1 and 3.2:.

1. Let e_1, e_2 and $\hat{e}_1, \hat{e}_2$ be two basis in the plane that are related by

$$\begin{cases} \hat{e}_1 & = 3e_1 + 2e_2 \\ \hat{e}_2 & = e_1 + e_2 \end{cases}$$

(a) Let the coordinates of a vector be $x = (x_1, x_2)$ in the (e_1, e_2) basis and $\hat{x} = (\hat{x}_1, \hat{x}_2)$ in the $(\hat{e}_1, \hat{e}_2)$ basis. Find a matrix S such that $x = S\hat{x}$ and a matrix Q such that $\hat{x} = Qx$.
(b) Let T be a linear operator on $\mathbf{R}^2$ and suppose that the matrix A for T in the (x_1, x_2) coordinates, i.e. in the e_1, e_2 basis, is

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$$

Find the matrix $\hat{A}$ for T in the $(\hat{x}_1, \hat{x}_2)$ coordinates, i.e. in the $\hat{e}_1, \hat{e}_2$ basis.

2. Consider the matrix

$$A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$$

(a) Find the eigenvalues and eigenvectors of the matrix.
(b) Diagonalize A, i.e. find a matrix Q such that $\hat{A} = QAQ^{-1}$ is diagonal.

3. Diagonalize the matrix

$$A = \begin{bmatrix} 1 & -4 & -3 \\ -3 & 2 & 3 \\ -4 & -4 & 2 \end{bmatrix}$$

4. Determine if the following matrices can be diagonalized with a real similarity transformation QAQ^{-1} where Q has real elements), and if so diagonalize them and if not explain why.

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad \begin{bmatrix} 3 & 0 \\ 2 & 3 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

When its not possible you are asked to actually prove that it is not possible!