Lecture 5: 2.1 Regular Surface.

Def $S \subset \mathbf{R}^3$ is a *regular surface*, if for each $p \in S$ there is a neighborhood V of p in $\mathbf{R}^3$ and a map $\mathbf{x} : U \to V \cap S$ of an open set U onto $V \cap S$ such that

1. $\mathbf{x}$ is differentiable.
2. $\mathbf{x}$ is a homeomorphism, i.e. continuous and one-to-one with continuous inverse.
3. For each $q \in U$ the differential $d\mathbf{x}_q$ is one-to-one.

Condition 3 is the regularity condition. The differential is the linear part of the map, whose definition we will come back to in the next lecture. For now this just means that

$$\mathbf{x}_u = \frac{\partial \mathbf{x}}{\partial u} = \Big(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\Big), \quad \text{and} \quad \mathbf{x}_v = \frac{\partial \mathbf{x}}{\partial v} = \Big(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\Big)$$

are linearly independent, which is equivalent to $\mathbf{x}_u \wedge \mathbf{x}_v \neq 0$. Condition 3 will guarantee the existence of a *tangent plane* at all points of S see Problem 2.2.3 in the book where its not satisfied.

Condition 2 will prevent the surface from self intersecting see problem 2.2.10 in the book where its not satisfied.

For an example of a parametrization see Example 2.2.1 in the book.