Math 150A Midterm 1, Fall 98 Lindblad.

1. Given a parameterized curve

$$\alpha(s) = \left(\frac{1}{3}(1+s)^{3/2}, \frac{1}{3}(1-s)^{3/2}, \frac{1}{\sqrt{2}}s \right)$$

Show that the parameter s is the arc length. Determine the curvature $k(s)$, the torsion $\tau(s)$ and the *Frenet trihedron* $\mathbf{t}(s)$, $\mathbf{n}(s)$, $\mathbf{b}(s)$.

2. Let $\alpha : I \subset \mathbf{R} \to \mathbf{R}^3$ be a regular curve parameterized by arc length s. Assume that the curvature $k(s) = \frac{1}{2}$ and the torsion $\tau(s) = 0$. What curve must α be?

3. Show that $S = \{(x, y, z) \in \mathbf{R}^3; \, xyz = 1\}$ is a regular surface, and find parameterizations whose coordinate neighborhoods cover it. Compute the tangent space $T_p(S)$ to $\mathcal{S}$ at the point $p = (1, 2, \frac{1}{2})$.

4. Let $S_1 = \{(x, y, z) \in \mathbf{R}^3; \, z = 0\}$ be the xy-plane and let $S_2 = \{(x, y, z) \in \mathbf{R}^3; \, z = x^2 + y^2\}$ be the paraboloid. Let $\varphi : S_1 \to S_2$ be the map sending $(x, y, 0) \in S_1$ to $(-x, 2y, x^2 + 4y^2) \in S_2$. Let p be the point $(1, 2, 0) \in S_1$ and w be the tangent vector $(3, -1, 0) \in T_p(S_1)$. Let $d\varphi_p : T_p(S_1) \to T_{\varphi(p)}(S_2)$ be the differential. Compute $d\varphi_p(w)$.