

**Math 150A Midterm 2, Fall 98 Lindblad.**

1. Find the Gaussian curvature and the mean curvature at all point for the surface  $z^2 = x^2 + y^2 + 1$ .
2. Find the principal curvatures and directions of the surface  $z = \frac{3}{2}x^2 - 2xy$  at  $p = (0, 0, 0)$ .
3. Show that a point  $p$  on a regular surface is umbilical if and only if there is a number  $k = k(p)$  such that  $e = kE$ ,  $f = kF$  and  $g = kG$  there. (Hint: Look at the matrix of  $d\mathbf{N}_p$ . ) Find the umbilical points, if any, on the graph of  $z = xy$ .
4. A bilinear form  $b : \mathbf{R}^2 \rightarrow \mathbf{R}$  is called indefinite if  $b(u, u)$  is both positive and negative, i.e. there is  $u$  and  $v$  such that  $b(u, u) > 0$  and  $b(v, v) < 0$ . The second fundamental form can be seen as a (symmetric) bilinear form on the tangent space. Make this statement rigorous. At which type of point (ie, planar, hyperbolic, elliptic, or umbilical) is the second fundamental form indefinite?