

Math 150A Midterm 2, Fall 98 Lindblad.

1. Find the Gaussian curvature and the mean curvature at all point for the surface $z^2 = x^2 + y^2 + 1$.
2. Find the principal curvatures and directions of the surface $z = \frac{3}{2}x^2 - 2xy$ at $p = (0, 0, 0)$.
3. Show that a point p on a regular surface is umbilical if and only if there is a number $k = k(p)$ such that $e = kE$, $f = kF$ and $g = kG$ there. (Hint: Look at the matrix of $d\mathbf{N}_p$.) Find the umbilical points, if any, on the graph of $z = xy$.
4. A bilinear form $b : \mathbf{R}^2 \rightarrow \mathbf{R}$ is called indefinite if $b(u, u)$ is both positive and negative, i.e. there is u and v such that $b(u, u) > 0$ and $b(v, v) < 0$. The second fundamental form can be seen as a (symmetric) bilinear form on the tangent space. Make this statement rigorous. At which type of point (ie, planar, hyperbolic, elliptic, or umbilical) is the second fundamental form indefinite?