

1. Consider the curve defined parametrically by  $x(t) = t^3 - 3t^2 - t$  and  $y(t) = 2t^3 - 3t^2 - 2t$ .
  - (a) (10 pts.) Find the point where the curve intersects itself.
  - (b) (10 pts.) Determine the equation of each line tangent to the curve at the point you found in part (a).
  
2. (20 pts.) Do the points  $(1,2,1)$ ,  $(2,-3,1)$ ,  $(-2,2,4)$  and  $(3,-3,0)$  all lie in the same plane? Explain.
  
3. Consider the polar curve  $r = e^{2\theta}$  for  $-\pi/2 \leq \theta \leq \pi/2$ .
  - (a) (10 pts.) Find the length of the curve.
  - (b) (10 pts.) Find the values of  $\theta$  that correspond to a point on the curve having a horizontal or vertical tangent line.
  
4. Consider the function  $f(x, y) = \sqrt{x} \cos y$ .
  - (a) (10 pts.) Calculate the rate of change of  $f(x, y)$  at the point  $(2, \pi/4)$  in the direction of the vector  $\langle 3, 4 \rangle$ .
  - (b) (10 pts.) Find the maximum rate of change of  $f(x, y)$  at the point  $(2, \pi/4)$  and the direction in which it occurs.
  
5. (25 pts.) Find all local and absolute maxima and minima of  $f(x, y) = x^2 - xy + y^2 - 3y + 2$  on the region bounded by the lines  $y = x$ ,  $y = -x$  and  $y = 3$ .
  
6. (20 pts.) Find the maximum and minimum of  $f(x, y, z) = xyz^2$  subject to the constraint  $5x^2 + 3y^2 + 2z^2 = 20$ .
  
7. Let  $\mathcal{D}$  denote the region of the  $xy$ -plane that is bounded by the curve  $x = y^2$  and the line  $y = 6 - x$ .
  - (a) (10 pts.) Calculate the area of  $\mathcal{D}$ .
  - (b) (10 pts.) Find the surface area of the portion of the plane  $2x + 2y - z = -1$  which lies directly above  $\mathcal{D}$ .
  
8.
  - (a) (10 pts.) Evaluate  $\iint_D e^{x^2+y^2} dA$  where  $D$  is the region in the  $xy$ -plane such that  $x^2 + y^2 \leq 1$ .
  - (b) (15 pts.) Find the volume of the solid region bounded by the parabolic cylinder  $x = 1 - y^2$  and the planes  $z + y = 2$ ,  $x = 0$  and  $z = 0$ .

9. (30 pts.) Circle your answer for each of the following questions.

(a) Two vectors  $u$  and  $v$  are perpendicular if

- i.  $u \times v = 0$       ii.  $u \cdot v = 1$       iii.  $u \cdot v = 0$       iv. none of the above

(b) Two vectors  $u$  and  $v$  are parallel if

- i.  $u \times v = 0$       ii.  $u \times v = 1$       iii.  $u \cdot v = 0$       iv. none of the above

(c) The vector  $u \times v$  is always perpendicular to

- i.  $v \times u$       ii.  $u \cdot v$       iii.  $(u \cdot v)u$       iv.  $(u - v) \times (u + v)$

(d) The point  $(0,1,1)$  in rectangular coordinates is the same as which of the following points given in cylindrical coordinates

- i.  $(\sqrt{2}, \pi/2, \pi/2)$       ii.  $(1, \pi/2, 1)$       iii.  $(\sqrt{2}, \pi/2, 1)$       iv.  $(1, \pi, 1)$

(e) The point  $(1,1,\sqrt{2})$  in rectangular coordinates is the same as which of the following points given in spherical coordinates

- i.  $(\sqrt{2}, \pi/4, \pi/4)$       ii.  $(\sqrt{3}, \pi/2, \pi/4)$       iii.  $(2, \pi/4, \pi/4)$       iv.  $(\sqrt{3}, \pi/4, \pi/2)$

(f) The surface  $x^2 = -1 - z^2 + y^2$  is a

- i. ellipsoid      iii. cone  
ii. hyperboloid of one sheet      iv. hyperboloid of two sheets

(g) The surface  $z^2 + x = y^2$  is a

- i. hyperbolic paraboloid      iii. hyperboloid of one sheet  
ii. ellipsoid      iv. parabolic cylinder

(h) Horizontal cross-sections of the surface  $x^2 = 3z^2 + 2y^2$  are most generally

- i. circles      ii. ellipses      iii. hyperbolas      iv. parabolas

(i) Vertical cross-sections of the surface  $z = y^2 - x^2$  are most generally

- i. circles      ii. ellipses      iii. hyperbolas      iv. parabolas

(j) The surface  $\rho \sin \phi (\cos \theta + 5 \sin \theta) + 2\rho \cos \phi = 1$  is a

- i. cylinder      ii. plane      iii. paraboloid      iv. hyperboloid