Math 21C Final Exam Lecture B Agler Winter 02

1. Use the cross product to calculate the area of the triangle with vertices $(1, 1, 1), (2, 3, 2)$, and $(3, -1, 4)$.

2. At what point do the curves $\vec{r_1}\,(t) = < t, t^2, t^3 >$ and $\vec{r_2}\,(t) = < 1 + t, 4t, 8t^2 >$ intersect? Find their angle of intersection to the nearest degree.

3. Find an equation for the planes consisting of all points that are equidistant from the points $(1, 2, 3)$ and $(-1, 1, -1)$.

4. For $0 \leq t \leq 1$ a particle moves with position vector given by $\vec{r}\,(t) = 2t^{3/2}\,\vec{i} + cos\,2t\,\vec{j} + \sin 2t\,\vec{k}$. Find the initial speed of the particle and the total distance it travels.

5. Find the points on the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$ where the tangent plane is parallel to the plane $z = x + y$.

6. Find and classify the critical points of $f(x, y) = x^4 - 8xy + 2y^2 - 3$.

7. A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the amount of cardboard used.

8. Find the volume of the solid bounded by the paraboloid $z = 10 - 3x^2 - 3y^2$ and the plane $z = 4$.

9. Find the area of the part of the surface $z = x + y^2$ that lies above the triangle with vertices $(0, 0), (1, 1)$, and $(0, 1)$.

10. Evaluate $\iiint\limits_E y\ dV$ where E is the solid tetrahedron with vertices $(0, 0, 0), (1, 0, 0), (0, 1, 0)$ and $(0, 0, 2)$.