Handout in Midterm 1, Chapter 10.-10.5,12.1-12.6:

Parametric equations for a curve: $x = f(t)$, $y = g(t)$, where $\alpha \le t \le \beta$.
Slope of tangent line $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$. Eq. of tanget line $y - y_0 = m(x - x_0)$.

Arc length: $L = \displaystyle\int_\alpha^\beta \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$.

Area of surface of revolution around the x-axis: $A = \displaystyle\int_\alpha^\beta 2\pi y \sqrt{(dx/dt)^2 + (dy/dt)^2}\, dt$.
Polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$, $\tan\theta = y/x$.

Chap. 12: Vectors If $P_0(x_0, y_0, z_0)$ and $P_1(x_1, y_1, z_1)$ are two points then the vector between them is $\overrightarrow{P_0P_1} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$.

Let $\mathbf{a} = \langle a_1, a_2, a_3\rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ be vectors and θ the angle between them.
Length: $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot product: $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$, $\quad \mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$.
$\mathbf{a}$ is parallel to $\mathbf{b}$ if $\mathbf{a} = c\mathbf{b}$, for some scalar c. $\mathbf{a}$ and $\mathbf{b}$ are perpendicular if $\mathbf{a}\cdot\mathbf{b} = 0$.

The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is the vector $\mathbf{proj_a}(\mathbf{b}) = \text{comp}_\mathbf{a}(\mathbf{b})\mathbf{u}$, where $\mathbf{u} = \mathbf{a}/|\mathbf{a}|$ is a unit vector in the direction of $\mathbf{a}$ and $\text{comp}_\mathbf{a}(\mathbf{b}) = \mathbf{b}\cdot\mathbf{u}$ is the component of $\mathbf{b}$ in the direction of $\mathbf{a}$.
Cross product: $\mathbf{a}\times\mathbf{b} = \langle a_2b_3 - a_3b_2\,,\ a_3b_1 - a_1b_3\,,\ a_1b_2 - a_2b_1\rangle$, if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k},$$

$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta$, $\quad \mathbf{a}\times\mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$.

The area of the triangle with $\mathbf{a}$ and $\mathbf{b}$ as two edges is $A = |\mathbf{a}\times\mathbf{b}|/2$.
The volume of the parallelepiped with $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ as three edges is $V = |\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$.
$\mathbf{a}\times\mathbf{b} = 0$ if $\mathbf{a}$ and $\mathbf{b}$ are parallel. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = 0$ if $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ lie in the same plane.

Parametric equations of a line: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$ where (x_0, y_0, z_0) is on the line and $\langle a, b, c\rangle$ is parallel to it: $\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$.

Equations of planes $ax + by + cz + d = 0$, where $\mathbf{n} = \langle a, b, c\rangle$ is a normal to the plane, i.e. it is perpendicular to it. If $P_0(x_0, y_0, z_0)$ is a point in the plane this can also be written as $\langle a, b, c\rangle\cdot\langle x - x_0, y - y_0, z - z_0\rangle = a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

The distance between a point $P_1(x_1, y_1, z_1)$ and the plane is $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

Examples of Quadratic surfaces: An Ellipsoid $x^2 + y^2 + 2z^2 = 1$, an Elliptic Paraboloid $z = x^2 + y^2$, a Hyperbolic Paraboloid $z = x^2 - y^2$, a Cone $z^2 = x^2 + y^2$ and a Hyperboloid $z^2 = x^2 + y^2 + 1$.