

**Lecture 1: Overview.** What is the course about?

Describing phenomenas in space, e.g. a particle moving along a curve in space, the temperature distribution in space or the gravitational field.

Understanding the geometry of space, e.g. curves and surfaces in space. Introducing three dimensional coordinate systems and measuring distances. We also use so called **vectors**, which are directed line segments, e.g the velocity vector of a particle or the gravitational field acting on a particle. Vectors can also be used to describe the geometry of planes.

Calculus for functions of several variables and for vectors. Defining the dot product and cross product of vectors. Finding max and min of functions of several variables. This involves defining the derivative of functions of several variables, so called directional derivatives and partial derivatives. Calculating areas and volumes which involves defining the integral of functions of several variables.

Chapter 10: Parametric equations of curves in the plane.

Chapter 12: Vectors and the geometry of space.

Chapter 13: Vector functions and space curves.

Chapter 14: Derivatives of functions of several variables.

Chapter 15: Integrals of functions of several variables.

**Section 10.1: Curves in the plane defined by parametric equations.** A curve can be defined by:

A graph

$$(10.1.1) \quad y = F(x), \quad a \leq x \leq b$$

(or  $x = G(y)$  ).

A Cartesian equation

$$(10.1.2) \quad h(x, y) = 0$$

**Parametric equations**

$$(10.1.3) \quad \begin{cases} x = f(t), \\ y = g(t) \end{cases}, \quad \text{where} \quad \alpha \leq t \leq \beta.$$

Think of  $t$  as the time and  $(x, y)$  as the position of particle at time  $t$ . The particle moves and traces out a curve.

**Ex. 10.1.1** The curve  $\begin{cases} x = \cos t, \\ y = \sin t \end{cases}$ , where  $0 \leq t \leq 2\pi$  is a parametrization of the circle, since  $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$ .

**Ex. 10.1.2** The curve  $\begin{cases} x = \cos 2t, \\ y = \sin 2t \end{cases}$ , where  $0 \leq t \leq \pi$  is another parametrization of the circle, since  $x^2 + y^2 = \cos^2(2t) + \sin^2(2t) = 1$ .

Note that both examples above describe the same curve. The difference is that the particle travels double as fast in the second example.