

Math 21C Midterm 1, Fall 02, Lindblad.

1. Consider the curve defined by the parametric equations

$$x = t, \quad y = 2t^{3/2}, \quad \text{where} \quad 0 \leq t \leq 2.$$

- (a) Write down the equation for the line that is tangent to the curve when $t = 1$.
- (b) Find the arc length of the curve.

2. Consider the curve C given by the parametric equations

$$x = \cos(t), \quad y = 2 + \sin(t), \quad \text{where} \quad 0 \leq t \leq 2\pi.$$

- (a) Find the points where the tangent to the curve is vertical.
- (b) Find the surface area obtained by rotating the curve around the x -axis.
(The surface is a donut.)

3. Let $\mathbf{a} = \langle 2, 2, 2 \rangle$ and $\mathbf{b} = \langle 5, c, 5 \rangle$ be vectors, where c is to be determined.

- (a) For what value of c is $\mathbf{b} = \langle 5, c, 5 \rangle$ orthogonal to $\mathbf{a} = \langle 2, 2, 2 \rangle$.
- (b) For what value of c is $\mathbf{b} = \langle 5, c, 5 \rangle$ parallel to $\mathbf{a} = \langle 2, 2, 2 \rangle$.

4. Let $P(1, 0, -3)$, $Q(0, -2, -4)$ and $R(4, 1, 6)$ be points.

- (a) Find the equation of the plane through the points P , Q and R .
- (b) Find the area of the triangle with vertices P , Q and R .

5. Consider the plane $x + 2y + z + 4 = 0$ and the point $P_1(1, 0, 1)$.

- (a) Find the equation of the line through P_1 that is perpendicular to the plane.
- (b) Find the point P in the plane that is closest to P_1 , i.e. such that the distance $|\overrightarrow{PP_1}|$ is the shortest distance between the plane and the point P_1 .