1. Consider the curve defined by the vector function $r(t) = \langle 6t+1, t^3, -3t^2 \rangle$.

   (a) (10 pts.) Find the equation of the line tangent to the curve at the point $(7, 1, -3)$.
   The tangent vector $r'(t) = \langle 6, 3t^2, -6t \rangle$ gives the direction of the tangent line at the corresponding point. Since the point $(7, 1, -3)$ corresponds to $t = 1$, the drection of the tangent line is given by $r'(1) = \langle 6, 3, -6 \rangle$. Thus the equation of the tangent line is

$$\begin{aligned} x(t) &= 7 + 6t \\ y(t) &= 1 + 3t \\ z(t) &= -3 - 6t \end{aligned}$$

   (b) (10 pts.) Find the length of the portion of the curve from the point $(-5, -1, -3)$ to $(7, 1, -3)$.
   Since the points $(-5, -1, -3)$ and $(7, 1, -3)$ correspond to $t = -1$ and $t = 1$, respectively, the arclength is given by

$$\begin{aligned} \int_{-1}^{1} |r'(t)|\ dt &= \int_{-1}^{1} \sqrt{36 + 9t^4 + 36t^2}\ dt \\ &= \int_{-1}^{1} \sqrt{9(t^2+2)^2}\ dt = \int_{-1}^{1} 3(t^2+2)\ dt = t^3 + 6t\Big|_{-1}^{1} = 14 \end{aligned}$$

2. Compute the following limits. If the limit does not exist, explain why.

   (a) (10 pts.) $\displaystyle\lim_{(x,y)\to(2,3)} \frac{x^2+xy+2y^2-1}{x^2-y^2+4} = \frac{4+6+18-1}{4-9+4} = -27.$
   For this limit we can simply plug in the values $x = 2$ and $y = 3$ because the rational function $\frac{x^2+xy+2y^2-1}{x^2-y^2+4}$ is defined at the point $(2, 3)$.

   (b) (10 pts.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+xy+y^2}$ does not exist.
   In this case, the limit does not exist because we get two different values if we take the limit along two different paths through the point $(0, 0)$. For example, if we let $y = 0$ then the limit becomes

$$\lim_{(x,0)\to(0,0)} \frac{x^2+0^2}{x^2+x0+0^2} = \lim_{x\to 0} \frac{x^2}{x^2} = 1.$$

   On the other hand, if we let $y = x$ then the limit becomes

$$\lim_{(x,x)\to(0,0)} \frac{x^2+x^2}{x^2+xx+x^2} = \lim_{x\to 0} \frac{2x^2}{3x^2} = 2/3.$$

3. Let $f(x, y, z) = e^{xy^2} + \ln(y + z^3)$. Compute the following partial derivatives of f.

   (a) (5 pts.) $f_x = y^2 e^{xy^2}$

   (b) (5 pts.) $f_y = 2xye^{xy^2} + \frac{1}{y+z^3}$

   (c) (5 pts.) $f_{zx} = f_{xz} = \frac{\partial}{\partial z} f_x = \frac{\partial}{\partial z}(y^2 e^{xy^2}) = 0$

(d) (5 pts.) $f_{zy} = f_{yz} = \frac{\partial}{\partial z} f_y = \frac{\partial}{\partial z}(2xye^{xy^2} + \frac{1}{y+z^3}) = \frac{-3z^2}{(y+z^3)^2}$

4. (20 pts.) Approximate $(\sqrt[3]{28})^2 + (\sqrt{24})^3$.
   Let $f(x,y) = (\sqrt[3]{x})^2 + (\sqrt{y})^3$. We will use the tangent plane

$$z = f(27,25) + f_x(27,25)(x-27) + f_y(27,25)(y-25)$$

to approximate $f(28,24)$. Note that we have chosen $(27,25)$ as our point of tangency since $f(27,25)$ can be easily calculated. While other points could be used, they would most certainly lead to a less accurate approximation.

$$f(27,25) = (\sqrt[3]{27})^2 + (\sqrt{25})^3 = 3^2 + 5^3 = 9 + 125 = 134$$
$$f_x(x,y) = \frac{2}{3}\frac{1}{\sqrt[3]{x}} \quad \Rightarrow \quad f_x(27,25) = \frac{2}{9}$$
$$f_y(x,y) = \frac{3}{2}\sqrt{y} \quad \Rightarrow \quad f_y(27,25) = \frac{15}{2}$$

Therefore,

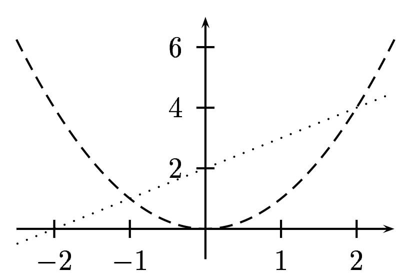
$$(\sqrt[3]{28})^2 + (\sqrt{24})^3 = f(28,24) \approx 134 + \frac{2}{9}(28-27) + \frac{15}{2}(24-25) = 134 - \frac{131}{18} = 126.7\bar{2}.$$

For comparison, the value given by a calculator is 126.7963802.

5. (20 pts.) Find all critical points of $f(x,y) = 2x^3 - 6xy + 3y^2 - 12y$. For each critical point, determine if it corresponds to a local minimum, local maximum, or saddle point.

$$f_x(x,y) = 6x^2 - 6y = 6(x^2 - y)$$
$$f_y(x,y) = -6x + 6y - 12 = 6(-x + y - 2)$$

Therefore, $f_x = 0$ when $y = x^2$ and $f_y = 0$ when $y = x + 2$. The points of intersection ocurr when $x^2 = x + 2$, in other words when $0 = x^2 - x - 2 = (x-2)(x+1)$. Therefore the only critical points are $(2,4)$ and $(-1,1)$.



Since $D = f_{xx}f_{yy} - (f_{xy})^2 = (12x)(6) - (-6)^2 = 36(2x-1)$, we have

$$D(2,4) = 36 \cdot 3 > 0 \quad \text{and} \quad f_{xx}(2,4) = 24 > 0$$
$$D(-1,1) = 36 \cdot -3 < 0$$

In other words, $f(2,4) = -32$ is a local minimum and $f(-1,1) = -5$ is a saddle point.