Geometric series divergent if $|r| \geq 1$, convergent for $|r| < 1$: $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

p-**series** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent if $p > 1$ and divergent for $p \leq 1$.

Divergence test If a_n does not tend to zero then $\sum a_n$ diverges.

Limit comparison test Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ exist and $0 < c < \infty$, then either both series converge or both diverge.

Absolute convergence test If $\sum |a_n|$ converges then $\sum a_n$ converges.

The Ratio Test

(i) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.

(ii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

(iii) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L = 1$, then the test is inconclusive.

For a given **power series** $\sum c_n (x-a)^n$ there are only three possibilities:
(i) The series converges only when $x = a$.
(ii) The series converges for all x.
(iii) There is a positive number R, called the **radius of convergence**,
such that the series is convergent if $|x-a| < R$ and divergent if $|x-a| > R$.

The **n-th degree Taylor polynomial of** f **at** a is given by

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

If $n = \infty$ it is called the Taylor series of f at a. If $a = 0$ it is called the Maclaurin polynomial or series. By Taylor's theorem

$$f(x) = T_n(x) + R_n(x),$$

where the remainder satisfies Taylor's inequality

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}, \qquad \text{for } |x-a| \leq d,$$

where M is a number such that $|f^{(n+1)}(x)| \leq M$, for $|x-a| \leq d$.