First order linear eq. $y' + p(t)y = g(t)$.
Multiply by integrating factor $\mu = e^{\int p\, dt}$ to get $\frac{d}{dt}\Big(y\mu\Big) = \mu g$.

Separable eq. $f(y)dy/dx = g(x)$
Formally multiply by dx: $f(y)dy = g(x)\,dx$ and integrate $\int f(y)\,dy = \int g(x)\,dx$.

Autonomous eq. $y' = f(y)$. Equilibrium points: $f(z) = 0$.
Stable if $\lim_{t\to\infty} y(t) = z$ for $y(0)$ sufficiently close to z. Unstable otherwise.

Homogeneous equations $ay'' + by' + cy = 0$
Characteristic equation $ar^2 + br + c = (r - r_1)(r - r_2) = 0$
Distinct real roots r_1 and r_2. Then $y = c_1e^{r_1t} + c_2e^{r_2t}$.
Complex conjugate roots $r_1, r_2 = \lambda \pm i\mu$. Then $y = Ae^{\lambda t}\cos(\mu t) + Be^{\lambda t}\sin(\mu t)$.
Repeated roots $r_1 = r_2$. Then $y = c_1e^{r_1t} + c_2te^{r_2t}$.

Undetermined coefficients To find a solution to $ay'' + by' + cy = g(t)$ try with:

$$g(t) = a_0t^n + ... + a_n, \qquad y(t) = t^s(A_0t^n + ... + A_n)$$

$$g(t) = (a_0t^n + ... + a_n)e^{\alpha t}, \qquad y(t) = t^s(A_0t^n + ... + A_n)e^{\alpha t}$$

$$g(t) = (a_0t^n + ... + a_n)e^{\alpha t}\cos\beta t + (b_0t^n + ... + b_n)e^{\alpha t}\sin\beta t, \qquad y(t) = t^s(A_0t^n + ... + A_n)e^{\alpha t}\cos\beta t + t^s(B_0t^n + ... + B_n)e^{\alpha t}\sin\beta t$$

where $s = 0,1,2$ is the smallest integer so y is not a solution to the homogeneous eq.

Variation of parameters To find a solution to $ay'' + by' + cy = g(t)$ try
$y = u_1y_1 + u_2y_2$, where y_1 and y_1 are solutions to the homogeneous eq. and
$u_1'y_1 + u_2'y_2 = 0$ and $u_1'y_1' + u_2'y_2' = g$.