First order linear eq. $y' + p(t)y = g(t)$.
Multiply by the integrating factor $\mu = e^{\int p\,dt}$ to get $\frac{d}{dt}(y\mu) = \mu g$ and integrate.
Separable eq. $f(y)dy/dx = g(x)$
Formally multiply by dx: $f(y)dy = g(x)\,dx$ and integrate $\int f(y)\,dy = \int g(x)\,dx$.

Autonomous eq. $y' = f(y)$. Equilibrium points: $f(z) = 0$.
Stable if $\lim_{t\to\infty} y(t) = z$ for $y(0)$ sufficiently close to z. Unstable otherwise.

Second order linear eq. with const. coeff. Homogeneous: $ay'' + by' + cy = 0$.
Characteristic equation $ar^2 + br + c = (r - r_1)(r - r_2) = 0$
Distinct roots r_1 and r_2. Then $y = c_1 e^{r_1 t} + c_2 e^{r_2 t}$.
Complex conjugate roots $r_1, r_2 = \lambda \pm i\mu$. Then $y = Ae^{\lambda t}\cos(\mu t) + Be^{\lambda t}\sin(\mu t)$.
Repeated roots $r_1 = r_2$. Then $y = c_1 e^{r_1 t} + c_2 t e^{r_2 t}$.

Undetermined coefficients To find a solution to $ay'' + by' + cy = g(t)$ try with:

$$\begin{aligned} g(t) &= a_0 t^n + ... + a_n, & y(t) &= t^s(A_0 t^n + ... + A_n) \\ g(t) &= (a_0 t^n + ... + a_n)e^{\alpha t}, & y(t) &= t^s(A_0 t^n + ... + A_n)e^{\alpha t} \\ g(t) &= (a_0 t^n + ... + a_n)e^{\alpha t}\cos\beta t & y(t) &= t^s(A_0 t^n + ... + A_n)e^{\alpha t}\cos\beta t \\ &\quad +(b_0 t^n + ... + b_n)e^{\alpha t}\sin\beta t, & &\quad + t^s(B_0 t^n + ... + B_n)e^{\alpha t}\sin\beta t \end{aligned}$$

where $s = 0,1,2$ is the smallest integer so y is not a solution to the homogeneous eq.

Laplace transform $\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0), \qquad \mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0),$$

$$\mathcal{L}\{(-t)^k f(t)\} = \frac{d^k}{ds^k}F(s), \qquad \mathcal{L}\{e^{bt}f(t)\} = F(s-b)$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}, \qquad \mathcal{L}\{te^{at}\} = \frac{1}{(s-a)^2}$$

$$\mathcal{L}\{e^{at}\sin bt\} = \frac{b}{(s-a)^2+b^2}, \qquad \mathcal{L}\{e^{at}\cos bt\} = \frac{s-a}{(s-a)^2+b^2}, \qquad \text{when } s > a.$$

Partial fractions Multiply up and equate the coefficients:

$$\frac{a_{1+k}s^{1+k} + \cdots + a_0}{\big((s-a)^2+b^2\big)(s-r)^k} = \frac{b_1 s + b_0}{(s-a)^2+b^2} + \frac{c_1}{s-r} + \cdots + \frac{c_k}{(s-r)^k}$$

First order linear systems: Homgeneous

$$\begin{aligned} x_1' &= a_{11}x_1 + a_{12}x_2 \\ x_2' &= a_{21}x_1 + a_{22}x_2 \end{aligned} \quad \Leftrightarrow \quad \mathbf{x}' = A\mathbf{x}, \quad \text{where} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Characteristic eq. $\det(A - rI) = \begin{vmatrix} a_{11} - r & a_{12} \\ a_{21} & a_{22} - r \end{vmatrix} = r^2 - (a_{11} + a_{22})r + a_{11}a_{22} - a_{12}a_{21} = (r - r_1)(r - r_2) = 0$.

Eigenvectors, for $i = 1, 2$: $A\mathbf{x}^{(i)} = r_i\mathbf{x}^{(i)} \Leftrightarrow (A - r_i I)\mathbf{x}^{(i)} = \begin{bmatrix} a_{11} - r_i & a_{12} \\ a_{21} & a_{22} - r_i \end{bmatrix} \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Distinct roots: $\mathbf{x} = c_1 e^{r_1 t}\mathbf{x}^{(1)} + c_2 e^{r_2 t}\mathbf{x}^{(2)}$.
Complex conjugate roots use $e^{(\lambda \pm i\mu)t} = e^{\lambda t}(\cos\mu t \pm i\sin\mu t)$ to get in real form.
Repeated roots: (i) If there are two nonparallel eigenvectors: $\mathbf{x} = c_1 e^{r_1 t}\mathbf{x}^{(1)} + c_2 e^{r_1 t}\mathbf{x}^{(2)}$.
(ii) If only one eigenvector $\mathbf{x}^{(1)}$ solve $(A - r_1 I)\mathbf{y} = \mathbf{x}^{(1)}$; $\mathbf{x} = c_1 e^{r_1 t}\mathbf{x}^{(1)} + c_2 e^{r_1 t}(t\mathbf{x}^{(1)} + \mathbf{y})$.

Undetermined coefficients Same as for second order eq. above but with the constants a_0, A_0 etc. replaced by constant vectors.