**Formulas for Final.** Space curve $C$: $\mathbf{R}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $a \le t \le b$.
Tangent vector: $\mathbf{R}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$.
Vector Field: $\mathbf{F}(x,y,z) = F_1(x,y,z)\mathbf{i} + F_2(x,y,z)\mathbf{j} + F_3(x,y,z)\mathbf{k}$. Scalar field $\phi$.

$\nabla\phi = \mathbf{grad}\ \phi = \phi_x\mathbf{i} + \phi_y\mathbf{j} + \phi_z\mathbf{k}$, $\nabla\cdot\mathbf{F} = \operatorname{div}\mathbf{F} = \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$, $\nabla\times\mathbf{F} = \mathbf{curl}\,\mathbf{F}$

$$\nabla\times\mathbf{F} = \mathbf{curl}\,\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_2 & F_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial z} \\ F_1 & F_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ F_1 & F_2 \end{vmatrix}\mathbf{k},$$

**Flow line:** $\dfrac{d\mathbf{R}(t)}{dt} = \beta\,\mathbf{F}\big(\mathbf{R}(t)\big)$, $\beta \ge 0$ or $\dfrac{dx}{F_1} = \dfrac{dy}{F_2} = \dfrac{dz}{F_3}$.

**Line Integral:** $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{R} = \int_a^b \mathbf{F}(\mathbf{R}(t))\cdot\mathbf{R}'(t)dt$

**The potential theorems** below hold under appropriate assumptions on the vector fields involved and the domains in which they are defined.
Theorem $\mathbf{F}$ is conservative, i.e.$\mathbf{F} = \nabla\phi$ if and only if it is irrotational, i.e.$\nabla\times\mathbf{F} = 0$.
Theorem $\mathbf{F} = \nabla\times\mathbf{G}$ for some $\mathbf{G}$ if and only if $\nabla\cdot\mathbf{F} = 0$.

**Change of variables:** $\begin{cases} x = x(u,v), \\ y = y(u,v) \end{cases}$, $dxdy = \left|\dfrac{\partial(x,y)}{\partial(u,v)}\right| dudv$, $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$

$$\begin{cases} x = x(u,v,w), \\ y = y(u,v,w), \\ z = z(u,v,w) \end{cases}, \quad dxdydz = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| dudvdw, \quad \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

**Surface area element $dS$ and unit normal $n$:**
Parametrized surface $x = x(u,v)$, $y = y(u,v)$, $z = z(u,v)$:
$dS = \left|\dfrac{\partial\mathbf{R}}{\partial u}\times\dfrac{\partial\mathbf{R}}{\partial v}\right| dudv$, $\mathbf{R}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$. $\mathbf{n} = \dfrac{\mathbf{N}}{|\mathbf{N}|}$, $\mathbf{N} = \dfrac{\partial\mathbf{R}}{\partial u}\times\dfrac{\partial\mathbf{R}}{\partial v}$.

Graph $z = f(x,y)$: $dS = \dfrac{dxdy}{|\cos\gamma|} = \dfrac{dxdy}{|\mathbf{n}\cdot\mathbf{k}|}$, $\mathbf{n} = \dfrac{\mathbf{N}}{|\mathbf{N}|}$, $\mathbf{N} = -f_x\mathbf{i} - f_y\mathbf{j} + \mathbf{k}$.

**The integral theorems** below hold under appropriate assumptions on the vector fields and domains involved and positive orientation of the boundaries and normals.
Divergence theorem $\displaystyle\iiint_V \nabla\cdot\mathbf{F}\,dV = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, $S$ is the boundary surface of $V$.
Stokes Theorem $\displaystyle\iint_S \nabla\times\mathbf{F}\ \cdot\mathbf{n}\,dS = \int_C \mathbf{F}\cdot d\mathbf{R}$, $C$ is the boundary curve of $S$.
Green's theorem $\displaystyle\iint_D \Big(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\Big)dxdy = \int_C Pdx + Qdy$, $C$ is the boundary of $D$.
Line integral of conservative field $\displaystyle\int_C \nabla\phi\cdot d\mathbf{R} = \phi(Q) - \phi(P)$, $Q$, $P$ are endpoints of $C$.
**Spherical coord.** $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$, $0 \le \theta < 2\pi$, $0 \le \phi < \pi$, $r > 0$. Volume $dV = r^2\,dr\sin\phi\,d\phi\,d\theta$. Surface area $dS = r^2\sin\phi\,d\phi\,d\theta$.