

Formulas for Midterm II.

Change of variables: $\begin{cases} x = x(u, v), \\ y = y(u, v) \end{cases}$, Jacobian $\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$.

$$\iint_D f(x, y) \, dx dy = \iint_{\bar{D}} f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, dudv,$$

$$\begin{cases} x = x(u, v, w), \\ y = y(u, v, w), \\ z = z(u, v, w) \end{cases}, \quad \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$\iiint_D f(x, y, z) \, dx dy dz = \iiint_{\bar{D}} f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| \, dudv dw,$$

Parametrized surface $\begin{cases} x = x(u, v) \\ y = y(u, v) \\ z = z(u, v) \end{cases}$.

Surface area $\iint_S dS = \iint_{\bar{D}} \left| \frac{\partial \mathbf{R}}{\partial u} \times \frac{\partial \mathbf{R}}{\partial v} \right| \, dudv$, where $\mathbf{R}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$.

Surface as a graph $z = f(x, y)$:

Surface area $\iint_S dS = \iint_D \frac{dx dy}{|\cos \gamma|} = \iint_D \frac{dx dy}{|\mathbf{n} \cdot \mathbf{k}|}$, where \mathbf{n} is the unit normal.

Vector Field: $\mathbf{F}(x, y, z) = F_1(x, y, z)\mathbf{i} + F_2(x, y, z)\mathbf{j} + F_3(x, y, z)\mathbf{k}$.

Flux $\iint_S \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{\bar{D}} \mathbf{F} \cdot \frac{\partial \mathbf{R}}{\partial u} \times \frac{\partial \mathbf{R}}{\partial v} \, du \, dv = \iint_D \mathbf{F} \cdot \mathbf{n} \frac{dx dy}{|\mathbf{n} \cdot \mathbf{k}|}$,

where \mathbf{n} is the exterior unit normal to the surface S .