Formulas for Midterm II.

Line Integral: $\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot \mathbf{c}'(t)dt = \int_{\mathbf{c}} F_1dx + F_2dy + F_3dz$

$$= \int_a^b \Big(F_1(x(t),y(t),z(t))\,\frac{dx}{dt} + F_2(x(t),y(t),z(t))\frac{dy}{dt} + F_3(x(t),y(t),z(t))\frac{dz}{dt}\Big)dt$$

Conservative fields: $\int_{\mathbf{c}} \nabla f\cdot d\mathbf{s} = \int_a^b \nabla f(\mathbf{c}(t))\cdot \mathbf{c}'(t)dt = f(\mathbf{c}(b)) - f(\mathbf{c}(a))$

Change of variables: $\begin{cases} x = x(u,v), \\ y = y(u,v) \end{cases}$, Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$.

$$\iint_D f(x,y)\,dxdy = \iint_{D^*} f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv,$$

$\begin{cases} x = x(u,v,w), \\ y = y(u,v,w), \\ z = z(u,v,w) \end{cases}$ Jacobian $\left|\dfrac{\partial(x,y,z)}{\partial(u,v,w)}\right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$

$$\iiint_D f(x,y,z)\,dxdydz = \iiint_{D^*} f\big(x(u,v,w),y(u,v,w),z(u,v,w)\big)\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| dudvdw,$$

Parameterized surface $\mathbf{T}(u,v) = x(u,v)\mathbf{i} + y(u,v)\mathbf{j} + z(u,v)\mathbf{k}$.

Surface area $\iint_S dS = \iint_{D^*} \left|\frac{\partial \mathbf{T}}{\partial u}\times\frac{\partial \mathbf{T}}{\partial v}\right| dudv$, where

Surface as a graph $z = f(x,y)$, with unit normal $\mathbf{n}$ making angle γ with $\mathbf{k}$.

Surface area $\iint_S dS = \iint_D \frac{dxdy}{|\cos\gamma|} = \iint_D \frac{dxdy}{|\mathbf{n}\cdot\mathbf{k}|}$.

Vector Field: $\mathbf{F}(x,y,z) = F_1(x,y,z)\mathbf{i} + F_2(x,y,z)\mathbf{j} + F_3(x,y,z)\mathbf{k}$.

Flux $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS = \iint_{D^*} \mathbf{F}\cdot\frac{\partial \mathbf{T}}{\partial u}\times\frac{\partial \mathbf{T}}{\partial v}\,du\,dv = \iint_D \mathbf{F}\cdot\mathbf{n}\frac{dxdy}{|\mathbf{n}\cdot\mathbf{k}|}$,

where $\mathbf{n}$ is the exterior unit normal to the surface S.

Exterior unit normal to parameterized surface: $\mathbf{n} = \pm\mathbf{T}_u\times\mathbf{T}_v/\|\mathbf{T}_u\times\mathbf{T}_v\|$,
to level surface $h(x,y,z)=0$: $\mathbf{n} = \pm\nabla h/\|\nabla h\|$, to graph set $h(x,y,z) = z - f(x,y)$.

Spherical coordinates $\mathbf{T} = \rho\sin\phi\,\cos\theta\,\mathbf{i} + \rho\sin\phi\,\sin\theta\,\mathbf{j} + \rho\cos\phi\,\mathbf{k}$, $0\le\phi\le\pi$, $0\le\theta\le 2\pi$.
$dS = \rho^2\,\sin\phi\,d\phi\,d\theta$, $dV = \rho^2\,\sin\phi\,d\rho\,d\phi d\theta$.

polar coordinates $x = r\cos\theta$, $y = r\sin\theta$, $r\ge 0$, $0\le\theta\le 2\pi$, $dxdy = rdrd\theta$.
Trigonometric formulas $\cos^2\theta = (\cos(2\theta)+1)/2$, $\sin^2\theta = (1-\cos(2\theta))/2$.