Formulas for Final. Space curve: $\mathbf{c}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $a \le t \le b$.
Tangent vector: $\mathbf{c}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j} + z'(t)\mathbf{k}$.
Vector Field: $\mathbf{F}(x,y,z) = F_1(x,y,z)\mathbf{i} + F_2(x,y,z)\mathbf{j} + F_3(x,y,z)\mathbf{k}$. Scalar field ϕ.

$$\nabla\phi = \mathbf{grad}\ \phi = \phi_x\mathbf{i}+\phi_y\mathbf{j}+\phi_z\mathbf{k}, \quad \nabla\cdot\mathbf{F} = \operatorname{div}\mathbf{F} = \frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}, \quad \nabla\times\mathbf{F} = \mathbf{curl}\,\mathbf{F}$$

$$\nabla\times\mathbf{F} = \mathbf{curl}\,\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} = \begin{vmatrix} \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_2 & F_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial z} \\ F_1 & F_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ F_1 & F_2 \end{vmatrix}\mathbf{k},$$

Flow line: $\dfrac{d\mathbf{c}(t)}{dt} = \beta\,\mathbf{F}\big(\mathbf{c}(t)\big), \quad \beta \ge 0$ or $\dfrac{dx}{F_1} = \dfrac{dy}{F_2} = \dfrac{dz}{F_3}$.

Line Integral: $\displaystyle\int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s} = \int_{\mathbf{c}} F_1dx + F_2dy + F_3dz = \int_a^b \mathbf{F}(\mathbf{c}(t))\cdot\mathbf{c}'(t)dt$

The potential theorems If $\mathbf{F}$ is C^1 in a simply connected domain then
Th 1: $\mathbf{F}$ is conservative, i.e.$\mathbf{F} = \nabla\phi$ if and only if it is irrotational, i.e.$\nabla\times\mathbf{F} = 0$.
Th 2: $\mathbf{F} = \nabla\times\mathbf{G}$ for some $\mathbf{G}$ if and only if $\nabla\cdot\mathbf{F} = 0$.

Change of variables:

$$\begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases}, \quad dxdy = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| dudv, \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix}$$

$$\begin{cases} x = x(u,v,w) \\ y = y(u,v,w), \\ z = z(u,v,w) \end{cases} \quad dxdydz = \left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| dudvdw, \quad \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

Surface area element dS and unit normal n:
Parameterized surface $x=x(u,v)$, $y=y(u,v)$, $z=z(u,v)$:

$$dS = \left|\frac{\partial\mathbf{T}}{\partial u}\times\frac{\partial\mathbf{T}}{\partial v}\right| dudv, \quad \mathbf{T}(u,v) = x(u,v)\mathbf{i}+y(u,v)\mathbf{j}+z(u,v)\mathbf{k}, \quad \mathbf{n} = \frac{\mathbf{N}}{|\mathbf{N}|}, \ \mathbf{N} = \frac{\partial\mathbf{T}}{\partial u}\times\frac{\partial\mathbf{T}}{\partial v}.$$

Graph $z=f(x,y)$: $dS = \dfrac{dxdy}{|\cos\gamma|} = \dfrac{dxdy}{|\mathbf{n}\cdot\mathbf{k}|}$, $\mathbf{n} = \dfrac{\mathbf{N}}{|\mathbf{N}|}$, $\mathbf{N} = \nabla G = -f_x\mathbf{i} - f_y\mathbf{j} + \mathbf{k}$, $G = z - f(x,y)$.

The integral theorems below hold under appropriate assumptions on the vector fields and domains involved and positive orientation of the boundaries and normals.

Gauss' theorem $\displaystyle\iiint_V \nabla\cdot\mathbf{F}\,dV = \iint_S \mathbf{F}\cdot\mathbf{n}\,dS$, S is the boundary surface of V.

Stokes Theorem $\displaystyle\iint_S \nabla\times\mathbf{F}\ \cdot\mathbf{n}\,dS = \int_{\mathbf{c}} \mathbf{F}\cdot d\mathbf{s}$, $\mathbf{c}$ is the boundary curve of S.

Green's theorem $\displaystyle\iint_D \Big(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\Big) dxdy = \int_{\mathbf{c}} Pdx + Qdy$, $\mathbf{c}$ is the boundary of D.

Line integral of conservative field $\displaystyle\int_{\mathbf{c}} \nabla\phi\cdot d\mathbf{s} = \phi(Q) - \phi(P)$, Q, P are endpoints of $\mathbf{c}$.

Spherical coord. $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$, $0 \le \theta < 2\pi$, $0 \le \phi < \pi$, $r > 0$. Volume $dV = r^2\,dr\sin\phi\,d\phi\,d\theta$. Surface area $dS = r^2\sin\phi\,d\phi\,d\theta$.