Math 20E Midterm 2, Spring 2000, Lindblad.

1. S be the part of the surface $z = x + y^2$ lying above the triangle $D = \{(x, y);\ 0 \le x \le y \le 1\}$ in the xy-plane.

(a) Write up the integral giving the surface area of S.

(b) Find the surface area of S, i.e. evaluate the integral in (a).

2. Let S be the part of the unit sphere $\{(x, y, z);\ x^2 + y^2 + z^2 = 1\}$ that lies inside the solid cone $\{(x, y, z);\ y \ge \sqrt{z^2 + x^2}\}$.

(a) Find the unit normal $\mathbf{n}$ to S oriented so $\mathbf{n} \cdot \mathbf{j} > 0$.

(b) Let $\mathbf{F} = \mathbf{i} + \mathbf{j}$. Find the Flux of the vectorfield $\mathbf{F}$ out from S: $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS$.

3. By making the change of variables $u = e^{-y} \cos x$, $v = e^{-y} \sin x$, calculate

$$\iint_R \frac{1}{\cos^2 x}\, dxdy$$

where R is the region in the xy plane given by

$$\frac{1}{4} \le e^{-y} \cos x \le \frac{1}{2}, \qquad \frac{1}{4} \le e^{-y} \sin x \le \frac{1}{2}.$$

4. A parametrization of the torus T is given

$$x = \cos\theta\,(4 + \cos\phi), \qquad y = \sin\theta\,(4 + \cos\phi), \qquad z = \sin\phi,$$

$$\text{where} \qquad 0 \le \theta \le 2\pi \qquad 0 \le \phi \le 2\pi.$$

(a) Calculate the surface area element dS in terms of $d\theta d\phi$ and use it to calculate the area of the surface of the torus T.

(b) Find the unit normal to the surface.