

Math 210B Final Winter 2000, March 24. 11.30-2.30 in HSS 1330, 2.30-4.15 in AP&M 7421. Leave the exam at the math. dep. front desk AP&M 7018. Please justify your answers carefully. For problems 1 and 4, I want you to use the method I ask for. There are alternative methods and if you have time you can use these to check your answer.

1. Use spherical harmonics to solve

$$\Delta u = 0, \quad \text{in } x^2 + y^2 + z^2 \leq 1, \quad u(x, y, z) = z^2, \quad \text{when } x^2 + y^2 + z^2 = 1$$

2. Prove that $\phi_n(x) \rightarrow \delta(x)$ if

$$\phi_n(x) = \frac{ne^{-n|x|}}{2}$$

3. Let $f(x)$ be the the period function defined by

$$f(x + 2\pi) = f(x), \quad \text{for all } x \text{ and } f(x) = |x|, \quad \text{when } |x| \leq \pi$$

(a) Find the Fourier coefficients $\{c_k\}$ for f .

Let $f_n(x) = \sum_{k=-n}^n c_k e^{ikx}$.

(b) Does $f_n \rightarrow f$ in $L^2[-\pi, \pi]$?

(c) Does $f_n \rightarrow f$ uniformly?

4. Let T_n be the polynomials of degree n defined by

$$(T_n, T_m) = \int_{-1}^1 T_n(x)T_m(x)(1-x^2)^{-1/2} dx = \delta_{mn}$$

(a) Use Gram-Schmidt's orthogonalization procedure to find T_0 , T_1 and T_2 .

(b) Express x^2 in the basis $\{T_n\}$.

To do this problem you have to make a trigonometric substitution in the integral and use the trigonometric double angle formulas. These formulas can be derived from Euler's formulas $\cos \theta = (e^{i\theta} + e^{-i\theta})/2$ and $\sin \theta = (e^{i\theta} - e^{-i\theta})/2i$.