MATH 257B TOPICS IN DIFFERENTIAL GEOMETRY-RELATIVITY

Lecture 1: Introduction.

1.2 Spacetime in prerelativity and Special relativity.
One of the difficulties in understanding relativity is overcoming our misconceptions about space and time from Newtonian physics.
In Newtonian physics we have a concept of an absolute time and for each fixed time the universe is a 3 dimensional space of simultaneous events; given an event p, all other events are either in the future of p, in the past of p, or simultaneous to p.
However in special relativity we learn that no information can pass faster then the speed of light and hence there is a future light cone whose interior consist of events in the future of p and a past light cone whose interior consist of events in the past of p, but everything outside the light cone is not casually related to p.

In special relativity as well as in prerelativity physics there is a notion of inertial motion, i.e. the "nonaccelerating motion without subject to external forces.
A basic postulate is that physics should look the same for all inertial observers.
If we have inertial observers O and O', moving apart with constant velocity in the x direction, that label the same event by (t,x,y,z) and (t',x',y',z'), then

$$t'=t,\quad x'=x-vt,\quad y'=y,\quad z'=z,$$

in prerelativity whereas in relativity they are related by a Lorentz transformation

$$t'=(t-vx)/(1-v^2)^{1/2},\quad x'=(x-vt)/(1-v^2)^{1/2},\quad y'=y,\quad z'=z$$

where we have chosen units so the speed of light $c=1$. In fact the Lorenz transformations are precisely the linear transformations that leave the light cones invariant, in other words the speed of light is the same in any inertial frame.

1.3 The spacetime metric.
In prerelativity physics the time interval between events $\triangle t$ as well the spacial interval $|\triangle \mathbf{x}|$ between simultaneous events have an absolute meaning.
In special relativity neither of these have absolute meaning and the only observer independent quantity is the Minkowski metric

$$-(\triangle t)^2+(\triangle x)^2+(\triangle y)^2+(\triangle z)^2$$

In fact this is the only quadratic form invariant under Lorentz transformations.

1.4 General Relativity and gravity.
Newton's second law states that mass times acceleration is equal to the force:

$$m\frac{d^2\mathbf{x}}{dt^2}=\mathbf{F}$$

Newton thought of gravity as a force, the force on a mass m by a larger mass M is

$$F=-\frac{GmM}{r^2}$$

where r is the distance between M and m.

Newton's theory of gravitation is not consistent with special relativity since it invokes the notion of instantaneous influence of one body on another.
Einstein's thought that since all bodies fall the same in a gravitational field it ought to be a property of spacetime itself. A freely falling body is the natural unforced state. It is the motion along a geodesic in the spacetime metric:

$$\frac{d^2x^i}{dt^2}+\Gamma^i_{jk}\frac{dx^j}{dt}\frac{dx^k}{dt}=0.$$