Lecture 7: 3.3 Gaussian coordinates and an interpretation of the curvature tensor. Recall the equation for a geodesic

$$\frac{d^2x^c}{dt^2} + \Gamma^c_{ab}\frac{dx^a}{dt}\frac{dx^b}{dt} = 0 \qquad \leftrightarrow \qquad T^a\nabla_a T^c = 0, \qquad T = \frac{dx^c}{dt}.$$

This system of differential equations have a unique solution for any initial position and velocity. This means that for each tangent vector $T \in V_p$ at a point p we have a geodesic $\gamma(t)$ through p in the direction of T. The exponential map is the map $V_p \ni T \to \gamma(1) \in M$, defined for small T. We can hence parameterize a neighborhood of $p \in M$ by a neighborhood of $0 \in V_p$. This is called geodesic normal coordinates.

The geodesics can also be used to define another type of coordinate system. Suppose that S is a space-like hyper-surface of codimension 1 in a Lorentzian manifold of dimension n. This means that $g_{ab}X^aX^b > 0$ for and tangent vector $X \in \widetilde{V}_p$ to the hyper-surface. Let n be the unit normal; $g_{ab}n^aX^b = 0$, $X \in \widetilde{V}_p$, $g_{ab}n^an^b = -1$. Let S be parameterized by local coordinates $(x^1, \ldots, x^{n-1})$ and consider the geodesics from a point on S when the parameter $t = 0$ and let S_t be the surface of all geodesics from different points at time t..... The coordinates $(x^1, \ldots, x^{n-1}, t)$ of M are called *Gaussian coordinates*........ see (3.3.6) in book.

We will now give an interpretation of the Riemannian curvature tensor as how much nearby geodesics accelerate from each other...... see (3.3.15)-(3.3.18) in book.