- Please put your name, ID number, and section number (or time) on your blue book.
- The exam is CLOSED BOOK, but you may use a page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

- If you email me your name, section and ID number, I'll reply with your grade.

1. (20 pts.) Evaluate the following. Remember to show your work!

   (a) $\lim_{x \to 0} \dfrac{e^x - 1}{\sin x}$ (b) $(1+i)^{30}$ where $i = \sqrt{-1}$

2. (60 pts.) Evaluate the following. Remember to show your work!

   (a) $\displaystyle\int \frac{2}{x^3 - x}\, dx$ (b) $\displaystyle\int_1^e \ln x\, dx$

   (c) $\displaystyle\int \sin^4 x \cos^3 x\, dx$ (d) $\displaystyle\int \frac{1}{t\sqrt{t^2-1}}\, dt$

3. (15 pts.) Find the three cube roots of $1 + i$. You may leave sines and cosines in your answer, but not inverse trig functions.

4. (20 pts.) The region bounded by $y^2 = 4x + 4$, $x = 8$, the x-axis and the y-axis is rotated about the x-axis. Write down integrals for the volume and surface area. **You do not need to evaluate the integrals.**

5. (15 pts.) Solve the differential equation $xe^{-t}\dfrac{dx}{dt} = t$ with the initial condition $x(0) = 1$.

6. (20 pts.) To estimate $\ln 2$, I plan to evaluate the integral $\int_1^2 \frac{dx}{x}$ using the Trapezoidal and Midpoint rules with $n = 10$.

   (a) One of these will give an overestimate for $\ln 2$ and the other will give an underestimate. Which gives which estimate and why?
   Without a reason, you will receive no credit.

   (b) I change my mind and decide to use $n = 20$. If E is the error using the Midpoint rule with $n = 10$, estimate the error using the Midpoint rule with $n = 20$. (Express the answer in terms of E.)

7. (15 pts.) The equation $r = 4 - \sin\theta$ describes a curve in polar coordinates which encloses a region. Draw a rough sketch of the region and write down an integral for its area.
   You do not need to evaluate the integral.

8. (35 pts.) According to a law of physics, water flows out of a hole in the bottom of a circular cylinder at a rate that is proportional to the square root of depth of the water; that is, the rate of outflow for depth y is $K\,y^{1/2}$ where K is some constant depending on the size of the hole.

   For a particular cylinder, water starts flowing out when the depth is 25 feet. After two minutes, the depth of water is 16 feet.

   (a) Using the above information, set up a differential equation for $y(t)$, the depth of the water in feet t minutes after the start of the flow. Express the 25 feet and 16 feet information as conditions on $y(t)$ for particular t.

   (b) **Derive** the formula $y(t) = (5 - t/2)^2$ by solving the differential equation.
   NOTE: "Derive" means you must **actually solve** the differential equation, not simply check that the given $y(t)$ is a solution.

   (c) What is the depth of water after 10 minutes? after 20 minutes?

END OF EXAM