1. (48 pts.) Evaluate the following. Remember to show your work!

   (a) $\lim_{x\to 0} \dfrac{\cos x - 1}{e^x - 1}$.

   A. This has the form 0/0, so apply l'Hospital's rule to get $\lim_{x\to 0} \dfrac{-\sin x}{e^x} = 0/1 = 0$. (You cannot apply l'Hospital's rule a second time.)

   (b) $F'(x)$ given that $F(x) = \int_{\sqrt{x}}^{2} \cos(t^2)\, dt$.

   A. $dF/dx = \big(dF/d\sqrt{x}\big)\big(d\sqrt{x}/dx\big) = -\cos\Big(\big(\sqrt{x}\big)^2\Big)(1/2\sqrt{x}) = -(\cos x)/2\sqrt{x}$.

   (c) [5.5, #27] $\int e^t\sqrt{1+e^t}\, dt$.

   A. Let $1 + e^t = u$ and so $e^t\, dt = du$. The integral becomes

$$\int u^{1/2}\, du = 2u^{3/2}/3 + C = 2(1+e^t)^{3/2}/3 + C.$$

   You could also have used the substitution $u = e^t$ followed by $1 + u = v$.

   (d) $\int_0^2 |x-1|\, dx$.

   A. This equals $\int_0^1 (1-x)\, dx + \int_1^2 (x-1)\, dx$, which equals 1 after a little bit of work. Alternatively, you could have looked at the graph and seen that the integral was the area of two right triangles with height and base 1.

2. (20 pts.) (a) Verify that $\ln|\sin u|$ is an antiderivative of $\cot u$.

   A. To verify the claim, it suffices to compute $d(\ln|\sin u|)/du$, which is a Math 20A problem: $d(\ln|\sin u|)/du = (d\sin u/du)/\sin u = \cos u/\sin u = \cot u$.

   (b) Compute $\int_{\pi/4}^{\pi/2} \cot x\, dx$. Your final answer may contain logarithms, but it should NOT contain trig functions.

   A. By (a) this is $\ln|\sin(\pi/2)| - \ln|\sin(\pi/4)| = \ln 1 - \ln(2^{-1/2}) = \ln(2^{1/2} = (\ln 2)/2$. Any of the forms for the logarithm is okay.

3. (12 pts.) [see 5.2 #49] Verify the inequality $\int_0^1 \sqrt{2+x^2}\, dx \le \sqrt{3}$ without evaluating the integral.

A. On the interval $[0,1]$ we have $2 + x^2 \le 3$ and so the integral is bounded above by $\int_0^1 \sqrt{3}\, dx = \sqrt{3}$.

4. (a) (15 pts.) [see 3.11#3] Given the table of information below, use a linear approximation to estimate $g(16)$.

x	0	5	10	15
$g(x)$	0	20	35	45

A. The two points closest to $x = 16$ are $(10, 35)$ and $(15, 45)$. The slope of the line through these points is $10/5 = 2$. Thus we predict $g(16) = g(15)+2(16-15) = 47$.

(b) (5 pts.) Do you think your prediction is an overestimate or underestimate? Why? You must give a reason to receive credit.

A. **Overestimate**. The line through $(0, 0)$ and $(5, 20)$ has slope 4. The line through $(5, 20)$ and $(10, 35)$ has slope 3. The line through $(10, 35)$ and $(15, 45)$ has slope 2. Since the slopes are decreasing, the curve appears to be concave down. A line tangent to such a curve lies above the curve. (You can see this easily if you draw a picture.)