I've noted if the problem or a near miss is in the text.

1. (10 pts.) Evaluate $\int_0^2 \sqrt{4-x^2}\,dx$ by interpreting it as an area.

A. (p.383, Ex.4) Squaring and rearranging $y = \sqrt{4-x^2}$ gives $x^2+y^2=4$, a circle of radius 2 centered at the origin. The integral is the area in the first quadrant and so equals $(\pi 2^2)/4 = \pi$.

Since the problem did not ask for an exact answer, you will receive credit for a reasonable numerical evaluation.

2. (30 pts.) Evaluate the following integrals using the tools discussed in the text.

$$\int (1-x)\sqrt{2x-x^2}\,dx \qquad\qquad \int_0^2 |\sin\,\pi x|\,dx.$$

A. (p.426, #26, #39) The substitution $u = 2x - x^2$ converts the first to $\int \frac{1}{2}u^{1/2}\,du = u^{3/2}/3 + C$ and so the answer is $(2x-x^2)^{3/2}/3 + C$.

The second integral equals $\int_0^1 \sin\pi x\,dx - \int_1^2 \sin\pi x\,dx$. The substitution $u = \pi x$ gives $\int \sin\pi x\,dx = (-\cos\pi x)/\pi + C$. Thus the answer is
$(-\cos\pi + \cos 0)/\pi - (-\cos\pi + \cos 2\pi)/\pi = 4/\pi$.

3. (30 pts.) Differentiate the functions

$$F(x) = \int_1^x \sqrt{1+u^4}\,du \qquad\qquad G(x) = \int_{x^2}^1 \ln(1-t^3)\,dt.$$

A. Both rely on the Fundamental Theorem of Calculus.

We have $F'(x) = \sqrt{1+x^4}$.

We have $G(x) = -\int_1^{x^2} \ln(1-t^3)\,dt$ and $dG/xd = (dG/dx^2)(d(x^2)/dx)$. Thus $G'(x) = -\ln(1-(x^2)^3)\,d(x^2)/dx = -2x\ln(1-x^6)$.

4. (30 pts.) Express the following as integrals. **DO NOT EVALUATE** the integrals. Sketches may be useful in obtaining partial credit if you make a mistake.

(a) The area bounded by the 3 curves

$$y = \sin(\pi x), \qquad y = x^2 - x \qquad \text{and} \qquad x = 2.$$

A. (p.428, #26) The first two curves intersect at $x = 1$ and the $\sin(\pi x)$ lies below $x^2 - x$ for $1 < x < 2$. Thus the answer is $\int_1^2 (x^2 - x - \sin(\pi x))\,dx$.

(b) (p.448 #9) The volume of the solid obtained by rotating the region bounded by the curves $y^2 = x$ and $x = 2y$ about the y-axis.

A. The curves intersect at the points $(0,0)$ and $(4,2)$. The answer is
$\int_0^2 \pi((2y)^2 - (y^2)^2)\,dy$.