

- Please put your name, ID number, and section number (or time) on your blue book.
- The exam is CLOSED BOOK, but you may use ONE SHEET OF NOTES.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (80 pts.) Evaluate the following integrals. Remember to show your work!

$$(a) \int_0^{1/2} \frac{x}{\sqrt{1-x^2}} dx \qquad (b) \int \sqrt{t} (t + \sqrt[3]{t}) dt$$

$$(c) \int \sin^2 t \cos^3 t dt \qquad (d) \int (\ln x)^2 dx$$

$$(e) \int_{-1}^1 \frac{1}{x^3} dx$$

2. (15 pts.) Set up the integral for the volume of the solid obtained by rotating the region bounded by

$$y = 0, \quad y = \sin x, \quad 0 \leq x \leq \pi$$

about the line  $y = 1$ .

Your answer should include a sketch of the region together with the line about which the region is being rotated.

3. (30 pts.) I have a function  $f(x)$  and know that

$$\begin{aligned} +4 &\leq f'(x) \leq +20 && \text{for } 0 \leq x \leq 5 && \text{and} \\ -3 &\leq f''(x) \leq -2 && \text{for } 0 \leq x \leq 5. \end{aligned}$$

I want to use either the Midpoint rule or the Trapezoidal rule to obtain a lower bound for  $\int_0^5 f(x) dx$ ; that is, an estimate which is smaller than  $\int_0^5 f(x) dx$ .

(a) Which should I use (Midpoint or Trapezoidal) and why?

*Hint:* A sketch of  $f(x)$  may help you find the answer.

(b) I would like guarantee that the error in my estimate is no larger than  $0.001 = 10^{-3}$ . How large must I make  $n$  to guarantee this?

**END OF EXAM**