Name: ______________________ **PID:** ______________

TA: ____________________ **Sec. No:** _____ **Sec. Time:** _____

Math 20B.
Final Examination
March 21, 2008

Turn off and put away your cell phone.
No calculators or any other devices are allowed on this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	8	
2	8	
3	6	
4	4	
5	6	
6	6	
7	6	
8	6	
Σ	50	

Questions 1 and 2 are True/False. For each statement, circle T if it is *always* True; circle F if it is *ever* False and leave it *blank* if you don't know. Each question is worth 8 points: a correct response earns 2 points, leaving it blank earns 1 point and an incorrect response earns 0 points.

1. (8 points) If R is the radius of convergence for the power series $\sum a_n x^n$, then:

   (**T** **F**) The series converges whenever $|x| < R$.

   (**T** **F**) The series converges whenever $|x| \leq R$.

   (**T** **F**) The series diverges whenever $|x| \geq R$.

   (**T** **F**) The series diverges whenever $|x| > R$.

2. (8 points)

   (**T** **F**) Suppose $A > 0$. When $\int \frac{x\,dx}{x^2+A}$ is evaluated, the answer will be a logarithm plus a constant of integration.

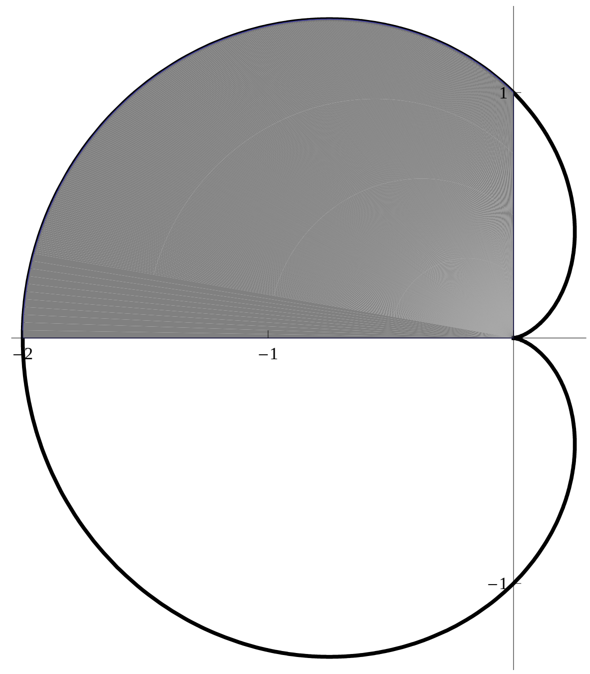
   (**T** **F**) Suppose $A < 0$. When $\int \frac{x\,dx}{x^2+A}$ is evaluated, the answer will be an arctangent plus a constant of integration.

   (**T** **F**) If $\sum a_n x^n$ has radius of convergence $R = 2$, then $\sum a_n (x-c)^n$ has radius of convergence $R = 2$ for every constant c.

   (**T** **F**) The series $\sum \frac{\sin(n)}{n^p}$ converges for $p > 1$.

3. (6 points) Write the complex number $(1+i)^{21}$ in the form $a+bi$. You need not simplify numbers like $2^{\frac{15}{2}}$. (Suggestion: you may wish to first put $1+i$ into polar form.)

4. (4 points) Write down an integral for the shaded portion of the cardioid $r = 1 - \cos(\theta)$ shown in the figure below. You need not evaluate the integral.



5. (6 points) Find the first four nonzero terms in the Taylor series about $x = 0$ (i.e., the Maclaurin series) for $f(x) = \sin(2x^3)$.

6. (6 points) Find the function $y(t)$ that satisfies the following differential equation with initial condition:

$$\begin{cases} (2-t)\frac{dy}{dt} - y = 0 \\ y(3) = -3 \end{cases}$$

7. Compute the following indefinite integrals.

(a) (3 points) $\int \sin(x^2)\cos(x^2)\, x\, dx$

(b) (3 points) $\int x^3 \ln(x)\, dx$

8. (6 points) Find the partial fraction expansion (PFE) of the following rational function:

$$f(x) = \frac{10x^2 + 2x - 6}{x^3 - x}$$