1. (20 pts) Suppose \((1 - x^2)^2 y'' + (1 - x) y' + (1 + x) y = 0\).
   (a) Determine the singular points. Which ones are regular?
   (b) Suppose we obtain a power series about \(x_0 = 0\) for \(y(x)\). Does the series converge at \(x = 1/2\)? You must give a correct reason for your answer to receive credit.

2. (15 pts) Compute the Laplace transform of \(y(t)\) given that
   \[ y''(t) + y(t) = 1 + 2e^t, \quad y(0) = 1 \quad \text{and} \quad y'(0) = 2. \]
   Note that you are not asked to find \(y(t)\).

3. (10 pts) A cylindrical tank is 100 feet high and has a circular base of diameter 50 feet. A hole in the bottom of the tank allows water to leak out according to Torricelli’s law: \(dh/dt = -5\sqrt{h}\), where \(h = h(t)\) is the depth of the water in feet and \(t\) is time in days. Water is pumped into the tank at a constant rate so that, if there were no hole, an empty tank would fill in 5 days. Write down a differential equation for \(h(t)\) when the tank starts out empty at \(t = 0\), taking into account the fact that the tank has a hole and water is being pumped in. Be sure to explain how you got the equation. You need not solve the equation.

4. (60 pts) Find the particular solutions to the following differential equations.
   (a) \(x y' = 3x - 2y, \quad y(1) = 2\).
   (b) \(x y' = (3x - 2)y, \quad y(1) = 2\).
   (c) \(y'' - y = t, \quad y(0) = y'(0) = 0\).
   (d) \(x^2 y' = x^2 - xy + y^2, \quad y(1) = 0\).

5. (20 pts) One solution to \(x^3 y'' + xy' - y = 0\) is \(y(x) = x\). Use reduction of order to find an independent solution for \(x > 0\).

6. (20 pts) Find the power series solution about \(x_0 = 0\) for the differential equation
   \[ (1 - x^2) y'' + 4y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -3. \]
   You may use the following fact to help you check your calculations, but you may not use it to find the solution: The answer is a polynomial of rather low degree.

7. (30 pts) (a) Find the general solution to \(2x^2 y'' + 3xy' - y = 0\) for \(x > 0\).
   (b) Find the general solution to \(2x^2 y'' + 3xy' - y = 9x^2\) for \(x > 0\).

That’s all folks!