

# MOOCs: An Inside View

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In 2012, as recently retired professors of mathematics (Linda) and economics (Rachel), we were attracted to the newly available—and free—massive open online courses (MOOCs), both for intellectual stimulation and to experience the latest innovation in teaching. Over the next year, we sampled dozens of courses in a variety of fields ranging from biology to cryptography. We completed all the coursework in some of them, earning “certificates” and even notations of distinction for our online efforts. In the process we added significantly to our knowledge in several fields related to our own and also gained perspective on the potential of MOOCs as a mode of instruction.

Discussing our observations with colleagues, we were surprised to learn that almost none of the many professors and administrators now involved in creating MOOCs or deciding on their appropriate role in their institution’s curriculum have actually “attended” even a single MOOC. Readers who have yet to discover the world of MOOCs might want to read the excellent Wikipedia article [1]. Then explore [www.coursera.org](http://www.coursera.org), [www.edx.org](http://www.edx.org), or [www.udacity.com](http://www.udacity.com), currently the most important US platforms for MOOCs, and peek into some courses. But just as peeking into a new textbook reveals only a bit about its value as a learning tool for our students, the same applies to MOOCs. We may be able to eliminate some MOOCs as unsuitable for our needs or those of our students on the basis of a quick peek, but evaluating others requires a larger investment of time.

To begin with the basics, a MOOC is usually an online presentation of all or part of a conventional college or graduate course, including syllabus, lectures, required or recommended reading, problem sets, and exams. Enrollment is free and easy, with no entrance requirements: A prospective student

need only establish an account with the relevant platform, e.g., Coursera, and then push the “Enroll” button for any course that is currently running or soon to begin. After agreeing to the honor code, the student can finally see what the course has to offer. Because a student must enroll first, this set-up tends to inflate the number of enrollees relative to the number who actually continue in the course, let alone complete it—thus helping to explain the often reported low rate at which students complete courses for which they have enrolled. The instructor of one recent Coursera course reported about 130,000 enrolled, of whom about 15,000 submitted at least one exam and about 9,000 completed the required coursework with an average grade high enough to qualify for a certificate.

Many MOOCs are shorter than most college courses—typically four to ten weeks—and accordingly cover less material. In contrast to the uniform lecture length of conventional courses, MOOCs usually consist of a much larger number of very short lecture segments. (Some research has shown that the average MOOC student finds even these short segments too long to sustain their interest. This is hardly a surprise to those of us with classroom experience.) Although most MOOCs follow a pre-announced schedule, students are free to watch a given week’s videos at their convenience during that week or later on. MOOC lecturers are typically research-oriented tenured faculty members at top colleges and universities, sometimes even winners of Nobel prizes or other academic distinctions.

Apart from length, MOOC lectures also vary in format. Most Coursera lectures have been recorded especially for this purpose, often in a way that makes the lecturer seem to be speaking directly to the MOOC student viewer. Lectures on edX are often recorded in front of a class, complete with students walking in front of the camera as they arrive late, coughing, chatting with neighbors, or answering questions posed by the lecturer. (We’ve noted some editing of these videos lately, perhaps in response to feedback from viewers.) In addition to their relative brevity, the online lectures in many courses contain pauses or mini-quizzes, during

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which the student has an opportunity to apply the most recent material and check for comprehension before proceeding further. Among the courses we have sampled, each week included a computer-graded homework assignment. Some courses also had peer-graded assignments. And courses often have one or more exams, again computer-graded. A final feature common to all the MOOCs we sampled is an online forum, which allows students to ask questions, answer questions posed by others, or simply to comment on their experience.

### Examples of “Successful” MOOCs

It is obvious to any experienced teacher that students learn in different ways. For concreteness, we describe here three of our more satisfying adventures in online learning, i.e., MOOCs that were successful for us. (As the small print in advertising for diet aids often cautions, your results may vary.) In the accompanying box, we suggest several other MOOCs that are likely to be of particular interest to mathematicians.

The first, “Financial Accounting,” was created by Brian Bushee, a professor at the University of Pennsylvania’s Wharton School. This sounded like a boring topic to one of us (the mathematician), but the economist argued that the subject is important in really understanding how corporations (and hence the world) actually operate. The mathematician grudgingly agreed to give it a try—and is glad she did. We already had Coursera accounts from previous courses, so we each logged in and pushed the “Enroll” button, one of us in San Diego and the other in Boston.

The course is noteworthy for several reasons. First, it turned out to be the best course we took, in our view a model of excellence in MOOC education. Second, financial accounting is a standard first-year M.B.A. course, and one of four first-year courses now being offered by Wharton as MOOCs. Third, the course, whether as a MOOC or in a classroom format, has no prerequisite beyond basic arithmetic, not even knowledge of Excel or basic programming. From the forum we knew that some enrolled students had studied accounting in the past and wanted to improve their knowledge, but most were new to the subject. Students lived in many different countries and represented a broad range of ages and prior educational attainments. We were far from the only retirees or Ph.D.’s, but many of our fellow students were undergraduates or even high schoolers in other countries, eager to sample elite US higher education in a physically and financially accessible form. Fourth, the course was indeed massive, with an enrollment of 130,290. Finally, the instructor provided extensive data and observations at the end of the course, so we knew more about this course than any other in which we had enrolled.



**Figure 1. Screenshot from Brian Bushee’s “Financial Accounting.”**

According to Bushee, who is a chaired professor of accounting and has received a variety of awards for excellence in teaching, his MOOC is closely modeled on the M.B.A. course he teaches at Wharton. Probably the most important difference is that the classroom version includes discussion of various cases (case analysis is a staple of M.B.A. education). The course ran for ten weeks and actually covered more material than the standard first M.B.A. course in accounting. Each week’s topic was covered in up to eight well-organized video segments, as much as three hours in total. The last part of each week’s offering applied the most recent material to the financial reports of an actual US company, 3M. The videos were created for the MOOC and show Bushee in his office rather than in a classroom. Students could also download the slides from each lecture or the full transcript—particularly valuable for students whose first language is not English.

There were nine homework assignments and two exams. These were in multiple-choice format but far from easy (at least for us), even though two tries were allowed on each assignment, and three tries on each exam. A very helpful feature was that the machine grading included explanations of the correct answers.

Students could use the online forum to raise and answer questions about the course material—the forum received an amazing 15,694 posts over ten weeks. Although Bushee noted in a “goodbye video” that most student questions to the forum were answered correctly by other students, this course was unusual in the degree to which Bushee himself replied to queries from the class. In other MOOCs, questions were answered more often by teaching assistants or, in many cases, no one except other students. But since both of us were enrolled in the course, we had a second way to interact. Our “bicoastal study group” of two met frequently on Skype to ponder some of the tricky questions on the homework assignments. This was especially important in a subject (like accounting or math) where each topic builds on prior material.

From our own experience in studying an unfamiliar subject, we appreciated that MOOCs are unlikely to be successful for the typical student unless supplemented by a nonmassive way for students to interact with each other and with someone (a professor or teaching assistant) willing and able to provide expert assistance.

One unique feature of this MOOC deserves mention. In lecturing to our own classes, we often raise questions that (we think, based on past experience) may be in the minds of students, and then provide answers. In “Financial Accounting,” Professor Bushee created cartoon virtual students who raised questions for him to answer and who also provided some comic relief to enliven what might otherwise have been a very dry presentation.

As academics, we were especially entertained by comments from a virtual dean who worried about alumni reaction to some critical remarks from Bushee about firms’ deceptive accounting practices.

A second MOOC on Coursera turned out to be a convenient way for a pure mathematician to learn a bit about machine learning, a heavily mathematical method in computer science that is now influencing our lives, from Netflix video recommendations

to self-driving cars. One of us (Linda) pushed the “Enroll” button for the course given by Andrew Ng, a Stanford computer science professor and one of the two founders of Coursera. The topics combined theory and practice, complete with an excellent introduction to Matlab and its open-source alternative, Octave. In the first week students learned the difference between supervised and unsupervised learning, and the course quickly got into mathematical methods. The weekly assignments included both multiple-choice quizzes on the lecture material and short programming assignments. Of course students could skip the included review of linear algebra, but Linda gained a new respect for matrices as convenient placeholders for manipulation of data. This MOOC provided an excellent ratio of knowledge gained to hours spent, as well as a handsome certificate!

### MOOCs for Mathematicians

For mathematicians interested in expanding their knowledge of subjects using tools from math, there are many other possible MOOCs to consider. For a “nuts and bolts” introduction to practical cryptography, try Coursera’s “Cryptography 1” given by Dan Boneh (Stanford). Some programming is required (and it’s fun when you finally manage to decode the cipher text), but there is also some interesting basic theory.

Want to know a bit about graph theory and applications to social networks? Try Coursera’s “Social and Economic Networks,” given by economics professor Matt Jackson (Stanford). That course covers many fascinating topics, including Erdős random networks, game theoretic modeling, and Bayesian learning, as well as special software for network analysis. Networks are currently a hot topic, and edX has now posted their own course, “Networks, Crowds, and Markets,” presented by three highly distinguished Cornell professors: theoretical computer scientists Eva Tardos (Fulkerson Prize) and Jon Kleinberg (Nevanlinna Prize), as well as economics professor David Easley. This multidisciplinary course may be less mathematical than others mentioned here, but the promised applications are plentiful.

Udacity’s flagship course, “Introduction to Artificial Intelligence” (AI), taught by Peter Norvig (Google) and Udacity co-founder Sebastian Thrun (Stanford), offers lessons in many techniques and their applications to artificial intelligence (robots, anyone?), including a thorough discussion of Bayes’ Law with many exercises. Unlike courses at Coursera and edX (and some later courses at Udacity), AI does not run on a set schedule. That is, students can sign up anytime and come back anytime. AI’s initial offering by Stanford in 2011 attracted more than 100,000 students from around the world and is often credited with starting the MOOC “revolution.”



Figure 2. Linda’s certificate from “Machine Learning.”

How about a course with actual theorems and proofs? Linda tuned into a six-week course on algorithms given by Stanford computer science professor Tim Roughgarden. Corresponding roughly to the first half of a junior-level course for computer science majors, the course introduced some basic algorithms for sorting and searching, with emphasis on estimation of performance time. The weekly multiple-choice quizzes contained clever problems concerning the theory, while the programming assignments required mainly straightforward implementations of the algorithms discussed. For the programming part, students could use any language and were graded on numerical answers rather than on the programs themselves.

“Algorithms” proved very satisfying and relevant for a mathematician who has taught undergraduate courses in probability and applied algebra. Understanding how sophisticated mathematics is used in related areas is certainly helpful in planning lectures that will hold students’ interest. Another benefit is in understanding and communicating with colleagues in other departments.

### Motivation and Evaluation

Could MOOCs play a direct role for students at a college or university? As retired academics, we were certainly not the target audience for the MOOCs in which we enrolled, though we found we had plenty of company from other older students, many with advanced degrees. Some friends wondered whether we would get academic credit for completing such courses. Our response, with plenty of old-fashioned degrees on our CVs: “Credit toward what?” But the potential to earn credit toward a degree, satisfy a prerequisite, or qualify for employment will be a crucial issue in the future role of MOOCs in higher education. We ourselves were motivated mostly by the opportunity to learn something new in a very convenient way at no cost, and also to see how other departments organize their courses. Although we were not interested in credit (we did like getting those nice certificates), the instant feedback from the quizzes, problem sets, and exams was useful in testing our understanding. While some of the “real” students on the MOOCs were mainly motivated by a desire to learn from world-renowned scholars, many others were eager to obtain a recognized credential. The latter students, much like the ones we have encountered in our classrooms, often seemed at least as interested in the course grade as the knowledge gained in the process of obtaining that grade. And, similar to our own students, they used the course forums to compare homework and exam scores, thus gauging how well they were doing relative to peers.

This brings us to the problem of evaluating what has been learned. As professors, we are only too familiar with the issues of evaluation in traditional

### The Master Method

If  $T(n) \leq aT\left(\frac{n}{b}\right) + O(n^d)$

then

$$T(n) = \begin{cases} O(n^d \log n) & \text{if } a = b^d \text{ (Case 1)} \\ O(n^d) & \text{if } a < b^d \text{ (Case 2)} \\ O(n^{\log_b a}) & \text{if } a > b^d \text{ (Case 3)} \end{cases}$$



**Figure 3. Screenshot from Tim Roughgarden’s “Algorithms.”**

courses. In large classes where it is easy to copy short answers from a neighbor’s exam, much time and effort are expended to prevent cheating, and the electronic age has added new problems to the old ones. For example, students’ cell phones may facilitate unauthorized collaboration during an in-class exam, while small computers or calculators can invert a matrix and evaluate a definite integral. So we often choose to ban all electronics from our exams, but thus limit our ability to evaluate how well students can use some important tools.

As MOOC students, we’ve experienced various ways to evaluate students’ work. The first and simplest is with multiple-choice questions, the effectiveness of which is often in question. (In our experience writing exam questions for GRE subject tests, we learned how hard it is to create a meaningful multiple-choice question.) In courses containing computations, another option is to ask for the final numerical result in a calculation. In our MOOC experience, machine grading sometimes rejected correct answers due to round-off errors. Obviously, human grading is also subject to error. For courses in which students must submit computer programs, MOOCs use software to test the submitted programs. A final and, to our minds, least satisfactory solution used in some MOOCs is peer review, in which each student must grade the work (essays or written explanations) of several other students. Of course it is difficult to evaluate the resulting evaluations, and the incentive for and ability of students to do this well is at best questionable. Imagine a complicated proof produced by a top student being graded by a weak student who barely knows what a proof is.

An informal method by which some MOOC students may seek to distinguish themselves and thereby attract positive attention from instructors even in a huge class is through frequent contributions to the MOOC forum, where they answer other students’ questions, provide explanations of subtle points, or post solutions to challenges. Some courses recognize the quality and quantity of forum contributions, and a record of exceptional work may later be used to help a student gain

entry into a graduate program or be considered for employment.

Regardless of the method by which student work is evaluated, a more basic issue is whose work is being evaluated. The problem is captured succinctly by the caption of Peter Steiner's 1993 cartoon in *The New Yorker*: "On the Internet, nobody knows you're a dog." To separate the dogs from the cats, in 2013 Coursera introduced a system that confirms the identity of the student. According to the announcement of this innovation, a student who opts for the "Signature Track" builds a Signature Profile that links the student's coursework to the student's identity. The profile includes a photo ID and a Signature Phrase, "a biometric profile" of the student's unique typing pattern. Unlike the courses themselves, enrollment in the Signature Track entails a payment for this additional service. A similar identity-verification scheme has also been introduced on edX.

Verifying student identity is a crucial element when coursework can be submitted for academic credit. In 2013, the American Council on Education (ACE) approved five Coursera courses taken on the Signature Track for academic credit, including three math courses. Students would submit their exams through an online monitoring service called ProctorU. However, it is still up to individual colleges to decide whether to accept these courses for credit or even as satisfying the prerequisite for a higher-level course. In recent years, many colleges have backed away from accepting the Educational Testing Service's Advanced Placement exam results for credit. But similar to APs, Coursera course results may be accepted to satisfy the prerequisites for higher-level courses.

### MOOCs for Math?

Any discussion of the potential for online learning specifically geared to math needs to begin with the Khan Academy, which describes itself as a "not-for-profit with the goal of changing education for the better by providing a free world-class education for anyone anywhere." It began in 2004 as the modest effort of Salman Khan to assist a young relative who needed help with math. The success of this distinctly nonmassive operation soon generated unmanageable demand from additional relatives and their friends. To extend his services to a much larger audience, Khan began to create very short YouTube tutorials on specific topics (primarily staples of middle-school and high-school math, such as converting fractions to decimals or finding square roots) for which students most often requested help. By now, almost all of these math-help videos have racked up hundreds of thousands of views—probably by math-challenged parents as well as their children—and in some cases more than a million. With financial support from foundations and individual donors, the Khan Academy

has been able to expand its staff as well as its subject coverage, though it is still far from achieving its goal of providing world-class education across a broad spectrum of subjects, and the treatment is still mainly geared to secondary school students. And some members of the mathematical community are less than enthusiastic about the Khan Academy math videos, believing that they do not promote real understanding.

It's noteworthy that Khan himself is not a mathematician and previously had no classroom teaching experience in the subject. He did earn a bachelor's degree in math at MIT, but his graduate work was in engineering and computer science; he also earned an M.B.A. at Harvard. Most members of the Khan Academy staff likewise have credentials in computer science rather than math. In this respect the Khan Academy effort is representative of what has happened so far with mathematics MOOCs. At this point only a few math-related MOOCs have been created by members of math departments. Among the few is the successful calculus MOOC created by Robert Ghrist (University of Pennsylvania). Ghrist says he was motivated by the opportunity to present the subject from his own point of view [2]. Likewise, Petra Bonfert-Taylor (Wesleyan) created a course in complex analysis, one of the few upper-level math courses currently offered as a MOOC. Rather than replicating a typical undergraduate course on complex analysis, she minimized course prerequisites and decided to forgo full rigor in order to "spark a lasting interest and curiosity in a beautiful corner of mathematics" [3].

One MOOC entrepreneur told us that many mathematicians react negatively to the idea of creating an online course in their subject. The reason is not clear. Do mathematicians lack the ego to be thrilled by the prospect of reaching 50,000 students in a single class? Or even 10,000? Would higher-level math MOOCs suffer because prerequisites or "mathematical maturity" are required for success? One of Linda's UCSD colleagues suggested that mathematics is too hard to be taught in a MOOC since students have no way to ask questions during the lectures. A similar sentiment was expressed in the "Musings" article by Harvey Diamond [3]. Could this disadvantage be offset by the opportunity for students to replay parts of MOOC lectures or overcome by having a live TA to ask?

In [4] AMS president Eric Friedlander expressed his skepticism about the introduction of MOOCs: "Who knows how that's going to influence mathematics teaching? Maybe one professor who normally teaches computer science will instead teach 500,000 students first-year calculus through a MOOC. Students don't learn that way, and we mathematicians have to say so." We find it interesting that Friedlander assumes the creator of a massive

calculus course would be a computer scientist rather than a mathematician. We are less certain than Friedlander about how students learn—surely they vary in how they learn—and we conjecture that MOOCs might be a tool that will help at least some students, whether in conjunction with a traditional course or as a substitute for it.

### How Should Math Departments Respond?

How should mathematics departments respond to the challenge posed by this new mode of teaching? As Friedlander's comment implies, mathematicians can't simply ignore MOOCs. What would happen if other departments, e.g., engineering and computer science, are able to prove that they can teach basic courses like calculus, differential equations, and linear algebra more effectively (and efficiently) using MOOCs? How about discrete math and probability?

So far the total MOOC experience for ordinary students in universities has been mixed. MOOCs created by Udacity for San Jose State University to teach remedial math courses and elementary statistics produced a low pass rate [5]. The issue of evaluation, discussed previously, need not be a deal breaker for MOOCs in mathematics, since exams could be written, proctored, and graded by humans, as they are now. However, the real problem may be that most students do not have the discipline and aptitude to go through a difficult course on their own without significant live human intervention. Udacity has subsequently entered into an agreement with Georgia Tech to create a fully accredited online M.S. in Computer Science. The new program will provide enrolled students with additional services, including individual coaching. Students will pay \$7,000, a fraction of the cost of the on-campus version of the degree, but will follow the same curriculum and receive the same degree as students in the traditional program. The courses themselves (minus the individual coaching) will be MOOCs, open to any interested student at no charge.

The idea of a flipped class, where students follow MOOC video lectures and take quizzes outside of class and discuss problems and questions in class, has been proposed and tried at a small number of institutions. A Yale math instructor [6] reported success in using a Coursera MOOC in an integral calculus class. Do math faculty really want to teach basic topics such as the techniques of integration over and over? Could it be better to have students watch a few lectures, replay where necessary, and then work on some problems in class? Already some students taking calculus in regular lecture classes report that they are watching "the guy from Penn" to see how he does it. MOOCs may also allow talented students to complete basic courses more speedily instead of being held back by their weaker classmates.

We all know that different mathematicians have different talents: some are problem solvers, some see hidden structures, some are quick to follow an argument or trend. (Of course there are those few who do it all better than anyone else!) It's likely that math students also have different kinds of mathematical talent, and the opportunity to experience a variety of different teaching methods, including (but not exclusively) MOOCs, might enable some to experience greater success than in standard classes, whatever their size.

### What MOOCs Can't Do

Many successful "products" of higher education credit a particular teacher for motivation, inspiration, or advice that helped them find the right path to achieving their goals, or perhaps even in determining what their goals might be. It is hard to believe that MOOCs can ever replace those personal encouraging words from professors who praise some students and urge others to think more. Can a MOOC write a letter of recommendation or give career advice? Sadly, today it is not mainly MOOCs but steadily increasing class size and increased use of adjunct faculty that threaten the student-teacher relationship that adds so much value to education.

Even where MOOCs are used simply to fill holes in a student's preparation, problems can arise (as in the San Jose State offerings of remedial math courses). When students are stuck, where can they find the help they need? Last year, Rachel taught a microeconomics course with a calculus prerequisite for entering students in a master's degree program. Students lacking the necessary preparation in calculus had been instructed to complete an online (non-MOOC) calculus course prior to the start of the micro course. But many of these students, all graduates of good colleges and universities in majors like political science or romance languages, got stalled in their efforts and did not complete the calculus course. With no one to ask for help when they got stuck, they were unable to move forward. As Linda's UCSD colleague emphasized, math is hard and students must be able to ask questions. In a MOOC, some questions can be answered via the online forum, but this is a pale substitute for the personal help that is (at least ideally) available to campus students. Can personal help be provided remotely? It will be interesting to see the results of Georgia Tech's approach, which pairs MOOCs with individual coaching.

Finally, what about the learning that comes from interaction among students? Can the online forum in a MOOC with students all around the globe replace the intellectual excitement of in-person discussions with fellow students? Here it is worthwhile to distinguish students by age and situation. An increasing proportion of today's students, both on campus and online, are

“nontraditional” students, usually older and more likely to have schedules constrained by family or work obligations. For these nontraditional students, the asynchronous nature of MOOCs is a major advantage, as is their availability from any location with reliable Internet access. No doubt these students would benefit from in-person discussions on campus, but for most that may not even be an option. The same is true for students of any age living far from the educational institutions that provide the MOOCs they “attend.” Perhaps this issue exemplifies the most obvious advantage and disadvantage of MOOCs: while online classes are extremely convenient for students to fit into their schedules and can be accessed remotely, this also means that MOOC students will not be in the same place at the same time as other humans, either fellow students or instructors.

In conclusion, we note again that the focus of this article is the potential for MOOCs in delivering instruction, and especially instruction in math and related fields. We have not tried to address the possible impact of MOOCs on university faculty with respect to their research mission. And finally, we should emphasize that the landscape of MOOCs is changing so rapidly that any published article, including this one, is already out of date in some respects.

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