Problem 1. Answer the following questions.

(i) Given a set \( S \subset \mathbb{R} \), give the definition of \( \sup S \).

(ii) Prove that \( \sup\{1 - \frac{1}{n} : n \in \mathbb{N}\} = 1 \).

(iii) Does the set \( \{1 - \frac{1}{n} : n \in \mathbb{N}\} \) have a minimum? Justify your answer.

Problem 2. Answer the following two questions.

(i) Define what it means for a sequence \((a_n)\) to diverge at \( +\infty \).

(ii) Show that the sequence \( \frac{n}{n+1} \) diverges at \( +\infty \).

Problem 3. Prove that \( \lim_{n \to \infty} \frac{n^3}{n^3 + 1} = 1 \) by

(i) using the definition of convergent sequence;

(ii) using the properties of limits of convergent sequences.

Problem 4. Prove that if \((a_n)\) and \((b_n)\) are convergent sequences, than \((a_nb_n)\) is a convergent sequence, and

\[
\lim_{n \to \infty} a_nb_n = \lim_{n \to \infty} a_n \lim_{n \to \infty} b_n.
\]

Bonus Problem. Fix \( c \in \mathbb{R} \). Suppose that \((a_n)\) is a sequence such that \( a_n = c \) for infinitely many indices \( n \). Prove that if \((a_n)\) converges, then \( \lim_{n \to \infty} a_n = c \).