Solution to Practice Midterm 1

(P1) P | Q | P → Q | P ∧ (Q → Ø) → Ø → (Ø → (Ø ∧ Q)) → (Ø → (Ø ∧ Q)) ∨ Q
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

P is only proof by contradiction
Q is only proof by contradiction

This is why proof works

This is the answer

(P2) Suppose by contradiction \( a, b \in \mathbb{Z} \) s.t. \( 2a + 30b = 1 \)

\[ 2 \cdot (70 + 10b) = 1 \implies 70 + 10b = \frac{1}{2} \text{ (integer)} \]

\[ 70 + 10b = \frac{1}{2} \]

\[ 3 \]
(P2) Proof by induction

**BS:** \( a_1 = 1, \quad a_2 = 1 + \frac{1}{1 + 1} = 1 + \frac{1}{2} = \frac{3}{2} > 1 = a_1 \)

**TS:** We need to prove the implication \( a_m > a_{m-1} \Rightarrow a_{m+1} > a_m \)

We have \( a_m > a_{m-1} \Rightarrow \frac{1}{a_m} < \frac{1}{a_{m-1}} \Rightarrow 1 + \frac{1}{a_m} < 1 + \frac{1}{a_{m-1}} \)

\[ \Rightarrow \frac{1 + \frac{1}{a_m}}{1 + \frac{1}{a_{m-1}}} > 1 + \frac{1}{a_{m-1}} \]

\[ \Rightarrow a_{m+1} > a_m \]

(\( P_2 \) Prove by contraposition. Suppose that \( m \) is not a multiple of 3.

By the hint, there is \( k \in \mathbb{Z} \) s.t. \( m = 3k + r \), \( r = 0, 1, 2 \).

If \( r = 0 \), then \( m = 3k \) is a multiple of 3, \( \mathbb{Z} \)

If \( r = 1 \), then \( m = (3k+1)^2 = 9k^2 + 6k + 1 = 3 (3k^2 + 2k) + 1 \)

\[ \Rightarrow m^2 \text{ is not a multiple of } 3 \]
If $n=2$, then $n^2 = (3k)^2 = 9k^2 + 12k + 4 = 3(3k^2 + 4k + 1) + 1$.

$\Rightarrow n^2$ is not a multiple of 3.