Noncommutative Algebra Conference
Abstracts

Jason Bell - Simon Fraser University

Automorphisms of Projective Varieties and Noetherian Algebras

Let X be a projective variety over a field of characteristic 0 and let σ be an automorphism of X. We show that if $x \in X$ and Y is a subvariety of X, then the set of integers n such that $\sigma^n(x) \in Y$ is a finite union of complete doubly-infinite arithmetic progressions along with a finite set. In particular, if the orbit of x under σ is not critically dense, then it is contained in a proper subvariety of X. This answers a question of Rogalski. We discuss the methods used to obtain this result as well as what goes wrong in positive characteristic. Finally, we relate this result to graded subalgebras of twisted homogeneous coordinate rings.

George Bergman - UC Berkeley

Who would have thought you could fit so much into it?

Let G be the group of all permutations of a countably infinite set.

It is straightforward that G contains a free group on countably many generators. A 1957 result of de Bruijn shows that it contains a free group on *continuum* many generators. His technique also shows that G contains the coproduct, in any variety $\mathbf{V}$ of groups, of continuum many copies of any countable group in $\mathbf{V}$, and, indeed, contains groups arising via a very broad class of generalizations of this construction. He also showed that G contains a coproduct in the variety of all groups (a.k.a. a free product) of continuum many copies of itself.

The analogous results hold for the endomorphism algebra A of a countably infinite dimensional vector space, for the monoid of endomaps of a countable set, and, except perhaps for the final assertion, for the lattice of equivalence relations on a countable set.

We also obtain some interesting *restrictions* on groups, associative algebras, and lattices embeddable in these objects.

Matyas Domokos - Renyi Insitute, Hungarian Academy of Sciences, Budapest

Multisymmetric Syzygies and the Commuting Variety

We introduce a very simple approach to the presentation of the algebra of multisymmetric functions, inspired by the theory of trace identities of matrices. This yields simultaneously the generators (first fundamental theorem) and the relations (second fundamental theorem) in an explicit form, under the assumption that the order of the symmetric group is invertible in the base field. As an application, we deduce a consequence relating the defining equations of the commuting variety. We indicate also some consequences for modular invariant theory of classical groups, and explain the construction (obtained with P. E. Frenkel) of some exotic vector invariants of the orthogonal group.

Daniel Farkas - Virginia Tech

Poisson Algebras and Their Polynomial Identities

Poisson polynomial identities appear implicitly in the same paper of Kostant that, in retrospect, introduced trace identities. We discuss the origins of Poisson PIs (actual and revisionist), illustrate their machinery using enveloping algebras and rings of differential operators, and publicize recent work of Giambruno, Mishchenko, Petrogradsky, and Regev.

Ed Formanek - Pennsylvania State University

Observations on the two-variable Jacobian Conjecture

I will review the work of Abhyankar and Zhang on the two-variable Jacobian Conjecture, and add a few observations.

Robert Guralnick -University of Southern California

Small Representations and Conjectures of Larsen and Serre

One class of algebras that has been studied in recent years are those in which all finite dimensional representations are completely reducible. We consider an analog of this for group algebras of finite and algebraic groups. Of course, one knows precisely when this holds – but we consider the problem of when small representations are completely reducible. It turns out that this comes down to studying representations of simple groups. There is a dichotomy – representations of Chevalley groups in the natural characteristic and representations when this is not the case. In the first case, one can use the extensive theory of representations of algebraic groups. We will focus on the second case and show how understanding the small representations relates to complete reducibility and leads to solutions of conjectures of Serre and Larsen.

S.K. Jain - Ohio University

Group Algebras: Injectivity and Other Conditions

If R is a self-injective ring then the following properties hold: (C1) Every complement right ideal $= eR$, $e = e^2$; (C2) If $aR \cong eR$, $e = e^2$, then $aR = fR$, $f = f^2$; (C3) If $eR \cap fR = (0)$, $e = e^2$, $f = f^2$, then $eR \oplus fR = gR$, $g = g^2$. Rings satisfying (C1) are called CS rings because of the property that complement is the same as summand. Rings satisfying (C1) and (C2) are von Neumann continuous rings. Rings satisfying (C1) and (C3) satisfy the property that every projection $\pi : A \oplus B \to A$, where A, B are right ideals, can be lifted to an R-homomorphism $\pi^* : R \to R$, and conversely. Such rings are known as π-injective. Let $R = KG$, be a group algebra and $T =$ torsion elements of G.

Lemma: (1) (Renaut et al) R is self-injective iff G is finite. (2) (Jain et. al) If R is π-injective then T is a locally finite normal subgroup. (3) (Jain et. al) If R is continuous then G is locally finite. (4) (Farkas) R is principally injective iff G is locally finite. If G is locally finite then KG need not be continuous (equivalently π-injective); for example, we may choose $G = \cup S_n$. We ask a question: Is a von Neumann regular continuous group algebra self-injective. We may remark that such a ring cannot be prime.

Theorem (Behn, J. Alg. 2000) Let $R = KG$ be a prime group algebra of a polycyclic-by-fnite group with characteristic of K different from 2. Then FAE: (1) KG is PP, (2) KG is CS, (3) G is either D_∞ or torsionfree. If $D \cong D_\infty$ then KG is a prime noetherian PI-ring. If G is torsionfree then KG need not be hereditary (Jain et. al, Proc AMS, 2000)

Theorem (Alahmadi-Jain-Srivasatava, Preprint): Let $R = KG$ be a semiprime CS group algebra of a polycyclic-by-finite group with characteristic of K different from 2 having no ring direct summands that are domains. Then KG is hereditary,

In case K is algebraically closed: $KG \cong KD_\infty \oplus M_m(KD_\infty), m \geq 2$.

Theorem (Beidar - Jain - Kunwar - Srivastava, J. Alg. 2003 and 2004): (1) A group algebra KG is local and CS iff $ChK = p$ and G is locally finite p-group, (2) Let KG local. Then $M_2(KG)$ is CS $\Leftrightarrow$ KG is self-injective (3) Let KG be semilocal. Suppose G is either locally finite or solvable. Then FAE (a) $M_n(KG), n > 1$, is CS, (b) $M_2(KG)$ is CS, (c) KG is self-injective, (d) G is finite.

Example: (Prime local continuous group algebra). Let p be a prime and $G = P_\infty = \cup_{n=1}^{\infty} P_n$

where for each n, P_n is a Sylow p-subgroup of S_{p^n} and $P_n \subset P_{n+1}$. Then G is a locally finite p-group and $\Delta^+(G) = 1$. Let K be a field of characteristip p. Then KG is a prime local continuous group algebra.

Following are some recent results on semilocal continuous group algebras (the first result is quite surprising and provides plenty of examples of continuous rings).

Theorem (1) Every commutative semilocal group algebra is continuous. (2) Every local PI group algebra is continuous (however, in general there exist PI-semiperfect group algebras which are not continuous), (3) Let $G = P \times H$ where P is an infinite locally finite p-group and H is a finite group with $p \nmid |H|$. Then KG continuous $\leftrightarrow$ H is abelian, (4) Let G be an infinite nilpotent group and K be a field of characteristic $p > 0$. Then FAE (a) KG is semiperfect continuous, (b) KG is semilocal continuous, (c) KG semiperfect CS, (d) KG semilocal CS, (e) $G \simeq P \times A$ where P is a locally finite p-group and A is a finite abelian group such that p does not divide the order of A.

Hanspeter Kraft - Mathematisches Institut, Universitaet Basel

Let G be a finite group and V a G-variety, i.e. an irreducible algebraic variety with a regular action of G. A compression of V is a G-equivariant dominant morphism $f : V \to X$ such that G acts faithfully on X. Some basic questions are: (a) How much can one compress a given action? (b) What are the incompressible G-varieties?

There is an interesting relation of this concept with the generic structure of Galois-coverings. E.g. given a Galois-covering $Y \to B$ with Galois group G, what are the largest closed subvarieties $A \subset B$ such that the covering is trivial over A?

Compressions of representations have been introduced by Buehler and Reichstein in order to study the number of parameters of equations of field extensions. They define the essential dimension of G which measures how much a linear representation of G can be compressed. Their main result is that the minimal number of parameters for an extension of degree n is equal to the essential dimension of the symmetric group S_n.

We will explain this relation and will give an explicit construction of a compression of S_n such that the image has dimension equal to $n - 3$ (for $n > 3$). Moreover, we show that the image is rational which implies that every field extension of degree n is defined over a purely transcendental extension of transcendence degree $n-3$ (for $n > 3$). The cases $n = 5$ and $n = 6$ are classical, due to Hermite and Joubert who constructed explicit equations for such extensions.

Susan Montgomery - University of Southern California

Stable Radicals and Semiprime Smash Products

In this talk I will discuss some recent results on three open questions concerning actions of Hopf algebra on algebras. For simplicity, assume that H is a finite-dimensional semisimple Hopf algebra over a field of characteristic 0, and that A is an H-module algebra.

(1) Is the Jacobson radical of A stable under H?

This question was raised by Bergman for graded algebras, that is for $H = (kG)^*$, in 1975.

(2) Is the prime radical of A stable under H?

(3) If A has no H-stable nilpotent ideals, is the smash product $A\#H$ semiprime?

This question was raised by Cohen and Fischman in 1986.

I will report on recent work with Linchenko and Small on these questions for PI-algebras.

Don Passman - University of Wisconsin, Madison

Filtrations in Semisimple Rings and Lie Algebras

This is joint work with Yiftach Barnea. We first describe the maximal bounded $\mathbb{Z}$-filtrations of Artinian semisimple rings. These turn out to be the filtrations associated to finite $\mathbb{Z}$-gradings. We also consider simple Artinian rings with involution, in characteristic not 2, and we determine those bounded $\mathbb{Z}$-filtrations that are maximal subject to being stable under the action of the involution.

We then move on to consider maximal $\mathbb{Z}$-filtrations of a complex semisimple Lie algebra L. Specifically, we show that if L is simple of type A_n, B_n, C_n or D_n, then these filtrations correspond uniquely to a precise set of linear functionals on its root space. We obtain partial, but not definitive, results in this direction for the remaining exceptional algebras. Maximal bounded filtrations were first introduced in the context of classifying the maximal graded subalgebras of affine Kac-Moody algebras, and the maximal graded subalgebras of loop toroidal Lie algebras. Indeed, our results complete this classification in most cases.

Finally, we take a closer look at the linear functionals on the root space which give rise to maximal filtrations. We show that they are related to certain semisimple subalgebras of L of full rank. In this way, we determine the "order" of these functionals and count them without the aid of computer computations. We also describe the associated graded Lie algebras of all of the maximal filtrations obtained in this manner.

Claudio Procesi - University of Rome

The Algebra of the Box-Spline

Formulas computing volumes and number of integral points, in certain variable polytopes associated to a finite list of vectors, are related to a standard function of numerical analysis, the box-spline.

We shall see how certain methods of commutative and non commutative algebra explain their qualitative behavior and furnish explicit formulas and computational algorithms.

J. Chris Robson - University of Leeds

Hereditary Noetherian Prime Rings

There has been substantial work done on hereditary Noetherian prime rings since the 1960s and this work has involved many ring theorists. The module theory resulting from this is surprisingly complete, yet non-trivial. It includes the existence of a complete set of invariants for finitely generated torsion-free modules of uniform dimension 2 or more. I aim to describe some of this and some other recent developments and open questions.

Murray Schacher - UC Los Angeles

We discuss several topics expressing the relationship between finite groups and finite dimensional division algebras.

Agata Smoktunowicz - University of Edinburgh/Polish Academy of Sciences

On finitely Generated Algebras with Gelfand-Kirillov Dimension 2

We mention some results and examples on affine algebras with finite Gelfand-Kirillov dimension. We concentrate on algebras with Gelfand-Kirillov dimension 2. We show that there are no graded (by natural numbers) algebras with Gelfand-Kirillov dimension strictly between 2 and 3. We will give some examples of algebras with small Gelfand-Kirillov dimension and show that centers in finitely generated domains with quadratic growth are affine.

J.T. Stafford - University of Michigan

Noncommutative Projective Surfaces

This is joint work with Dan Rogalski.

Noncommutative projective geometry uses the techniques and intuition of classical algebraic geometry to understand noncommutative connected graded algebras $A = A_0 + A_1 + \cdots$ and the resulting noncommutative projective schemes. In this talk we will show that these techniques can be used to completely determine the noncommutative surfaces that are birational to commutative surfaces. More precisely, suppose that the given algebra A is a noetherian domain, generated in degree one, with graded quotient ring of the form $Q(A) = k(X)[z, z^{-1}, \sigma]$, where σ is an automorphism of the projective surface X. Then we show that A is completely determined as a "naive" blow-up (in the sense of Keeler and Rogalski) of a scheme birational to X. This also allows one to give a very detailed description of the representation theory of A.

James Zhang - University of Washington

Homological identities over Iwasawa algebras

We present the Auslander-Buchsbaum formula and the Bass formula and dual versions of these for modules over noncommutative Iwasawa algebras. This is a joint work with Quanshui Wu.