Chapter 23

**Definition:** (1) A positive integer $n$ is said to be prime when $n > 1$ and the only positive divisor of $n$ are 1 and $n$.

(2) If an integer $n > 1$ is not prime then it is said to be composite.

**Theorem:** (Fundamental Theorem of arithmetic) Every positive integer greater than 1 can be written uniquely as a product of prime numbers with the prime factors in the product written in non-decreasing order.

**Theorem:** There are infinite many prime numbers.

Proof: We will prove by contradiction. Suppose that these are finite many prime numbers. Write them as: $p_1, p_2, \ldots, p_N$.

Let $m = p_1, \ldots, p_{N+1}$. By the fundamental Theorem of arithmetic, $m$ can be written as a product of prime numbers. So $m$ is divisible by some prime $p_i$, i.e., $p_i | m$. But we also have $p_i | p_1 \cdots p_{N+1}$. Thus $p_i | 1$. This is a contradiction. Hence there are infinitely many prime numbers.

Eg. (1) How many distinct prime factors does 72 have?

(2) How many distinct positive factors does 72 have?

(3) How many distinct factors does 72 have?

Solution: $72 = 2^3 \times 3^2$

(1) 2
(2) $4 \times 3 = 12$
(3) $2 \times 12 = 24$