Practice Problems for Final Exam

1. A function \( f : A \to B \) is injective if and only if the following holds.

"For every pair of elements \( a_1, a_2 \in A, f(a_1) \neq f(a_2) \)."

Write what it means for a function to not be injective.

2. Use induction to prove that \( 57 | (7^{n+1} + 8^{2n-1}) \) for any \( n \in \mathbb{Z}^+ \).

3. (a). State the Schröder-Bernstein theorem.

(b). Use it to show that \( (-2, -1) \cup (0, \infty) \) and \( \mathbb{R} \) are equipotent.

4. (a). State the inclusion-exclusion principle for three sets.

(b). From 1 to 500, how many integers are divisible by either 3 or 7?

5. Find \( \text{gcd}(516,564) \).

6. Find all integer solutions to the equation

\[ 516m + 564n = 6432. \]

7. (a). Show that for each \( j \in \mathbb{Z}^+ \), \( 10^j \equiv 1 \mod 9 \).

(b). Let \( n \) be a positive integer that is written in the decimal notation:

\[ n = a_ka_{k-1} \cdots a_1a_0 \]

where every \( a_j \) satisfies \( 0 \leq a_j \leq 9 \). Prove that

\[ n \equiv a_k + a_{k-1} + \cdots + a_0 \mod 9. \]

(c). How many solutions does this following equation?

\[ 9m + 18n = 123456789. \]

8. Solve the linear congruence \( 255x \equiv 15 \mod 621 \).

9. (a). Let \( a \in \mathbb{Z} \). Prove that \( 5 | a^2 \) if and only if \( 5 | a \).

(b). Prove \( \sqrt{5} \) is not a rational number.

10. All homework problems (both submitted and suggested) and examples (and exercises) in lecture notes.