Practice Problems for Final Exam

1. A function \( f : A \to B \) is injective if and only if the following holds.

   "For every pair of elements \( a_1, a_2 \in A, f(a_1) \neq f(a_2) \)."

   Write what it means for a function to not be injective.

2. Use induction to prove that \( 57 | (7^{n+1} + 8^{2n-1}) \) for any \( n \in \mathbb{Z}^+ \).

3. (a). State the Schröder-Bernstein theorem.

   (b). Use it to show that \((-2, -1) \cup (0, \infty)\) and \( \mathbb{R} \) are equipotent.

4. (a). State the inclusion-exclusion principle for three sets.

   (b). From 1 to 500, how many integers are divisible by either 3 or 7?

5. Find \( \gcd(230381, 222503) \).

6. Find all integer solutions to the equation

   \[ 516m + 564n = 6432. \]

7. (a). Show by induction that for each \( j \in \mathbb{Z}^+ \), \( 10^j \equiv 1 \mod 9 \).

   (b). Let \( n \) be a positive integer that is written in the decimal notation:

   \[ n = a_k a_{k-1} \cdots a_1 a_0 \]

   where every \( a_j \) satisfies \( 0 \leq a_j \leq 9 \). Prove that

   \[ n \equiv a_k + a_{k-1} + \cdots + a_0 \mod 9. \]

8. Solve the linear congruence \( 255x \equiv 15 \mod 621 \).

9. (a). Let \( a \in \mathbb{Z} \). Prove that \( 5 | a^2 \) if and only if \( 5 | a \).

   (b). Prove \( \sqrt{5} \) is not a rational number.

10. (a). State the fundamental theorem of arithmetic.

    (b). How many distinct prime factors does 72 have? How many distinct positive factors does 72 have?

11. All homework problems (both submitted and suggested) and examples(and exercises) in lecture notes.