Practice Problems for Midterm 1

1. Prove the following two statements are equivalent:
   (i) $P \Rightarrow Q$
   (ii) $(P \lor Q) \iff Q$

   **Hint:** Show they have the same truth table.

2. Let $S_n = \sum_{j=1}^{n}(-1)^{j+1}j^2$, for $n \geq 1$. Use induction to prove that $S_n = (-1)^{n+1} \frac{n(n+1)}{2}$.

3. Let $A$ be a subset of $\mathbb{R}$. Find the negation of the following statement:
   
   *For any $x \in \mathbb{R}$, there is $y \in A$, such that $y \geq x$.*

4. Prove the following statement:

   Let $n \in \mathbb{Z}$. If $n^{100} - 1$ is even, then $n$ is odd.

5. Prove that there do not exist integers $m$ and $n$ such that $27m + 18n = 3$.

6. Determine whether the following statement is True or False. If true, give a proof. If false, give a counterexample.

   *For any sets $X$ and $Y$, we have that $\mathcal{P}(X \cup Y) = \mathcal{P}(X) \cup \mathcal{P}(Y)$.*

7. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

8. All homework problems and examples (and exercises) in lecture notes.