Practice Problems

1. Compute $\log(i^3)$ and $\log(1 + i)$.

2. Write the definition of principle value of $z^c$ where $c \in \mathbb{C}$. Use the definition to compute $\frac{1}{i\pi}$.

3. Write the definition of harmonic functions.

4. (a). Let $u(x, y) = 2xy$. Prove $u$ is harmonic.
    
    (b). Find the harmonic conjugate of $u$.

5. (a). Compute $\int_C \frac{1}{z} dz$, where $C$ is the circle $|z - 1| = 2$ (counterclockwise oriented).

    (b). Compute $\int_C \frac{1}{(z-2)^2}$, where $C$ is any closed contour which does not pass through the point 2.

    (c). Compute $\int_C \cos(\frac{z}{2}) dz$, where $C$ is a contour from $z = 0$ to $\pi + 2i$.

    (d). Compute $\int_C \frac{z+2}{\pi} dz$, where $C$ is the contour $z = e^{i\theta}$ : $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$. 