Pratice Problems

1. Let $x, y \in \mathbb{R}$. Set $z = \frac{x-iy}{x+iy}$. Prove $|z| = 1$.

2. Find all cube roots of $i$ and write them in the rectangular form $x + iy$.

3. Let $S$ be the horizontal line in $\mathbb{C}$: $S = \{\text{Im}(z) = 1\}$. Determine the image of $S$ under the map $w = f(z) = \frac{i}{z}$.

4. Let $f(z) = |z|^2$.
   (a) Show that $f$ is differentiable at $z = 0$.
   (b) Show that $f$ is not differentiable at any point $z \neq 0$. 