Pratice Problems for Midterm II

Note: No books, notes, cheat sheets, calculator or any electronic devices are allowed during Midterm II exam.

Tips:

- You are required memorize(not limited to):

  Cauchy-Riemann equations in $xy-$coordinates, and the formula $f' = u_x + iv_x$;

  Definition of harmonic functions, Laplace equation, harmonic conjugate;

  Definition of exponential functions, logarithmic functions, power functions, sine and cosine functions

- You don't need to memorize the Cauchy-Riemann equations in polar form. If it will be needed in the exam, we will provide it.

1. Write $z = x + iy$ with $x, y \in \mathbb{R}$. Let $f(z) = x^2 + iy^2$. Find all points where f is complex differentiable and find the value of f' at these points. Then find where f is analytic. Show your work.

2. Let $u(x, y) = 2xy - x$.

   (a) Prove u is harmonic on $\mathbb{R}^2$.

   (b) Find all harmonic conjugates v of u on $\mathbb{R}^2$..

   (c) Let v be your answer in (b). Find $f(z)$ such that $f = u + iv$. Express the function f in terms of z. Show your work.

3. State the definitions of $\mathrm{Log} z$ and $\log z$. Then find $\log e$ and $\mathrm{Log}(-ei)$. Show your work.

4. Does $\mathrm{Log}(i^3)$ equal to $3\mathrm{Log} i$? Justify your answer.

5. Let $c \in \mathbb{C}$. State the definitions of z^c and P.V.z^c. Then find $\frac{1}{i^{2i}}$. Show your work.

6. State the definitions of $\sin z$ and $\cos z$. Then use the definitions to prove $\cos z = \sin(z + \frac{\pi}{2})$.