

$$\begin{aligned} 1(c) \quad \overline{(2+i)^2} &= \overline{(2^2 + i^2 + 4i)} \\ &= \overline{3+4i} \\ &= 3-4i \end{aligned}$$

$$\begin{aligned} (d) \quad |(2\bar{z}+5)(\sqrt{2}-i)| &= |(2\bar{z}+5)| |\sqrt{2}-i| \\ &= |2\bar{z}+5| \sqrt{\sqrt{2}^2+(-1)^2} \\ &= \sqrt{3} |2\bar{z}+5| \end{aligned}$$

$$\begin{aligned} 7. \quad |\operatorname{Re}(2+\bar{z}+z^3)| &\leq |2+\bar{z}+z^3| \\ &\leq |2| + |\bar{z}| + |z^3| \\ &= 2 + |z| + |z|^2 \\ &\leq 2 + 1 + 1 = 4 \end{aligned}$$

$$\begin{aligned}
 1 \text{ (a)} \quad \arg\left(\frac{-2}{1+\sqrt{3}i}\right) &= \arg(-2) - \arg(1+\sqrt{3}i) \\
 &= +\pi - \frac{\pi}{3} + 2k\pi \\
 &= \frac{2\pi}{3} + 2k\pi
 \end{aligned}$$

$$\text{Since } -\pi < \frac{2\pi}{3} \leq \pi, \quad \text{Arg}\left(\frac{-2}{1+\sqrt{3}i}\right) = \frac{2\pi}{3}$$

$$\begin{aligned}
 5 \text{ (c)} \quad (\sqrt{3}+i)^6 &= (2e^{i\pi/6})^6 \\
 &= 2^6 e^{i\pi} \quad (\text{De Moivre}) \\
 &= -64
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad (1+\sqrt{3}i)^{-10} &= (2e^{i\pi/3})^{-10} \\
 &= 2^{-10} e^{-i10\pi/3} \\
 &= 2^{-10} e^{-i3\pi} e^{-i\pi/3} \\
 &= -2^{-10} (1-\sqrt{3}i) \\
 &= 2^{-10} (-1+\sqrt{3}i)
 \end{aligned}$$

$$6. \quad \text{Re } z_1, \text{Re } z_2 > 0$$

$$\begin{aligned}
 \Rightarrow \quad -\frac{\pi}{2} &< \text{Arg } z_1 < \frac{\pi}{2} \\
 -\frac{\pi}{2} &< \text{Arg } z_2 < \frac{\pi}{2}
 \end{aligned}$$

$$\therefore -\pi < \text{Arg } z_1 + \text{Arg } z_2 < \pi$$

$$\begin{aligned}
 \text{But} \quad \arg z_1 z_2 &= \arg z_1 + \arg z_2 \\
 &= \text{Arg } z_1 + \text{Arg } z_2 + 2\pi k
 \end{aligned}$$

$$\therefore \text{Arg } z_1 z_2 = \text{Arg } z_1 + \text{Arg } z_2 + 2\pi k$$

But $-\pi < \text{Arg } z_1 + \text{Arg } z_2 < \pi$

$$\Rightarrow \text{Arg } z_1 z_2 = \text{Arg } z_1 + \text{Arg } z_2$$

$$\begin{aligned} 10(b) \quad \sin 3\theta &= \text{Im } e^{i3\theta} \\ &= \text{Im } (e^{i\theta})^3 \\ &= \text{Im } (\cos \theta + i \sin \theta)^3 \\ &= \text{Im } (\cos^3 \theta + 3\cos^2 \theta i \sin \theta + 3\cos \theta (i \sin \theta)^2 + (i \sin \theta)^3) \\ &= 3\cos^2 \theta \sin \theta - \sin^3 \theta \end{aligned}$$