1 (c) $\overline{(2+i)^2} = \overline{(2^2 + i^2 + 4i)}$

$= \overline{3+4i}$

$= 3-4i$

(d) $|(2\bar{z}+5)(\sqrt{2}-i)| = |(2\bar{z}+5)||\sqrt{2}-i|$

$= |\overline{2\bar{z}+5}| \sqrt{\sqrt{2}^2+(-1)^2}$

$= \sqrt{3}|2z+5|$

7. $|Re(2+\bar{z}+z^3)| \leq |2+\bar{z}+z^3|$

$\leq |2| + |\bar{z}| + |z^3|$

$= 2 + |z| + |z|^2$

$\leq 2 + 1 + 1 = 4$

1 (a) $\arg\left(\frac{-2}{1+\sqrt{3}i}\right) = \arg(-2) - \arg(1+\sqrt{3}i)$

$= +\pi - \frac{\pi}{3} + 2k\pi$

$= \frac{2\pi}{3} + 2k\pi$

Since $-\pi < \frac{2\pi}{3} \leq \pi$, $\operatorname{Arg}\left(\frac{-2}{1+\sqrt{3}i}\right) = \frac{2\pi}{3}$

5 (c) $(\sqrt{3}+i)^6 = (2e^{i\pi/6})^6$

$= 2^6 e^{i\pi}$ (De Moivre)

$= -64$

(d) $(1+\sqrt{3}i)^{-10} = (2e^{i\pi/3})^{-10}$

$= 2^{-10} e^{-i\,10\pi/3}$

$= 2^{-10} e^{-i3\pi} e^{-i\pi/3}$

$= -2^{-10}(1-\sqrt{3}i)$

$= 2^{-10}(-1+\sqrt{3}i)$

6. $\operatorname{Re} z_1, \operatorname{Re} z_2 > 0$

$\Rightarrow -\frac{\pi}{2} < \operatorname{Arg} z_1 < \frac{\pi}{2}$

$-\frac{\pi}{2} < \operatorname{Arg} z_2 < \frac{\pi}{2}$

$\therefore \quad -\pi < \operatorname{Arg} z_1 + \operatorname{Arg} z_2 < \pi$

But $\arg z_1 z_2 = \arg z_1 + \arg z_2$

$= \operatorname{Arg} z_1 + \operatorname{Arg} z_2 + 2\pi k$

$\therefore \quad \operatorname{Arg} z_1 z_2 = \operatorname{Arg} z_1 + \operatorname{Arg} z_2 + 2\pi k$

But $-\pi < \operatorname{Arg} z_1 + \operatorname{Arg} z_2 < \pi$

$\Rightarrow \quad \operatorname{Arg} z_1 z_2 = \operatorname{Arg} z_1 + \operatorname{Arg} z_2$

10(b) $\quad \sin 3\theta = \operatorname{Im} e^{i3\theta}$

$= \operatorname{Im} (e^{i\theta})^3$

$= \operatorname{Im} (\cos\theta + i\sin\theta)^3$

$= \operatorname{Im} (\cos^3\theta + 3\cos^2\theta \, i\sin\theta + 3\cos\theta \, (i\sin\theta)^2 + (i\sin\theta)^3)$

$= 3\cos^2\theta \sin\theta - \sin^3\theta$