1. $-8-8\sqrt{3}\,i = 16\,e^{i\,4\pi/3}$

$$= 16\,e^{i(4\pi/3 + 2k\pi)} \qquad k = 0, \pm 1, \pm 2, \ldots$$

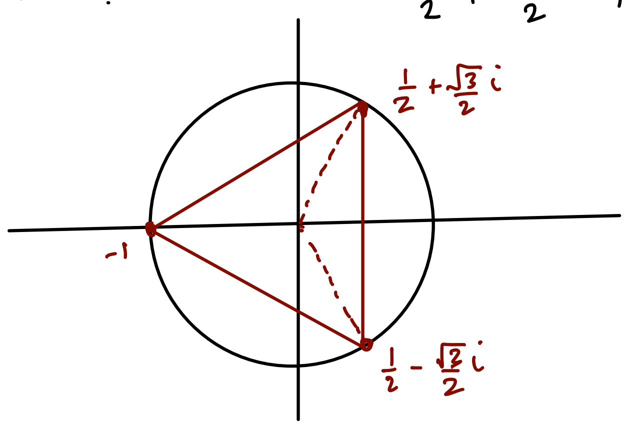
$$\therefore\ (-8-8\sqrt{3}\,i)^{1/4} = 2\,e^{i(\pi/3 + \frac{k\pi}{2})} \qquad k = 0, 1, 2, 3$$

Thus the three roots are $\pm(1+\sqrt{3}\,i)$ for $k = 0, 2$
$\pm(\sqrt{3}-i)$ for $k = 1, 3$

2. $-1 = e^{i\pi} = e^{i(\pi + 2k\pi)} \qquad k = 0, \pm 1, \pm 2, \ldots$

$$\therefore\ (-1)^{1/3} = e^{i(\pi/3 + \frac{2k\pi}{3})} \qquad k = 0, 1, 2$$

$\therefore$ The three roots are $\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -1,\ \frac{1}{2} - \frac{\sqrt{3}}{2}i$



3. $z^4 + 4 = 0$

$$\Rightarrow\ z^4 = -4$$

$$\Rightarrow\ z^4 = 4e^{i\pi} = 4e^{i(\pi + 2k\pi)}$$

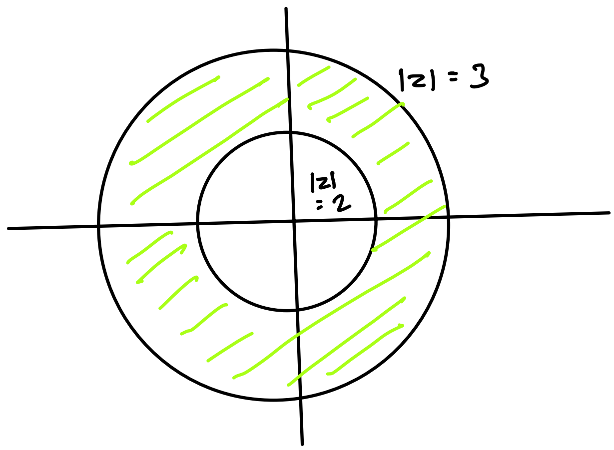
$$\Rightarrow\ z = \sqrt{2}\,e^{i(\pi/4 + k\pi/2)}$$

$$\therefore\ z = 1 \pm i,\ -1 \pm i$$

$$\therefore\ z^4+4 = \overbrace{(z-1-i)(z-1+i)}\overbrace{(z+1-i)(z+1+i)}$$

$$= (z^2 - 2z + 2)(z^2 + 2z + 2)$$

4. A



for (a) $|w| > 3$, the set $\{ |z-w| < \frac{|w|-3}{2} \} \subseteq S^c$

(b) $|w| < 2$, the set $\{ |z-w| < \frac{2-|w|}{2} \} \subseteq S^c$

(c) $|w| = 2$ and any neighbourhood $\{ |z-w| < \delta \}$,

$w + \frac{\delta w}{2|w|} \in S$ $\quad$ $w - \frac{\delta w}{2|w|} \in S^c$

(d) $|w| = 3$ is similar to (c)

(e) for $w \in S$ $\{ |z-w| < \frac{1}{2} \min \{ |w|-2, 3-|w| \} \} \subseteq S$

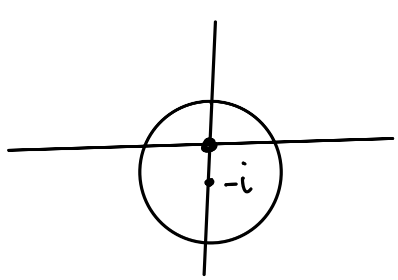
$\therefore$ The exterior is given by $\{ w \mid |w| > 3, |w| < 2 \}$

interior is given by S itself

boundary is given by $\{|z| = 3\} \cup \{|z| = 2\}$

thus S is open

B $S = \{ |z+i| = 2 \} \cup \{0\}$



for $w \notin S$ $\quad \{ |z-w| < \frac{1}{2} \min\{ ||w+i|-2|, |w| \} \} \subseteq S^c$

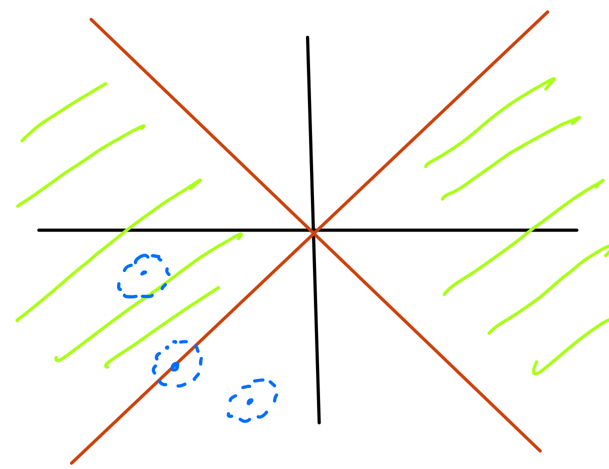
for $w \in S$ and $\{ |z-w| < \delta \}$,

$$w \pm \frac{\delta}{2} \frac{w}{|w|} \in S^c \quad \text{but} \quad w \in S$$

$\therefore$ The interior is empty,
exterior is S^c,
boundary is S.

Thus S is closed.

5. $S = \{ Re(z^2) > 0 \}$



Let $z = x + iy$

$\therefore \quad Re(z^2) = x^2 - y^2 > 0$

$\therefore$ The closure is given by
$Re(z^2) \geq 0$