1. $f(z) = \frac{\bar{z}^2}{z} = \frac{\bar{z}^3}{z\bar{z}} = \frac{\bar{z}^3}{|z|^2}$

Setting $z = x + iy$

$f(z) = \frac{(x-iy)^3}{x^2+y^2}$

$= \frac{x^3 - 3x^2iy - 3xy^2 + iy^3}{x^2+y^2}$

$= \frac{x^2 - 3xy^2}{x^2+y^2} + i\, \frac{y^3 - 3x^2y}{x^2+y^2}$

2. $f(z) = z + \frac{1}{z}$

writing $z = re^{i\theta}$

$f(z) = re^{i\theta} + \frac{1}{r}e^{-i\theta}$

$= r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta)$

$= \left(r + \frac{1}{r}\right)\cos\theta + i\left(r - \frac{1}{r}\right)\sin\theta$

3. $w = z^2$

writing $w = u + iv$ $\quad z = x + iy$

$u + iv = (x+iy)^2 = x^2 - y^2 + 2ixy$

$\therefore\ u = x^2 - y^2$

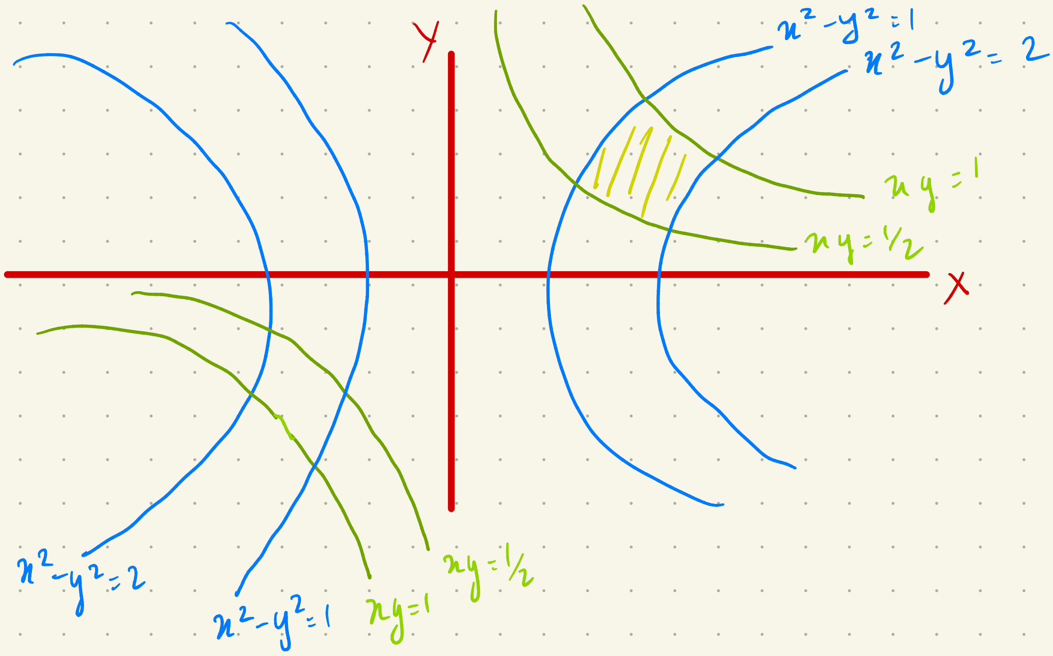
$v = 2xy$

$1 < u < 2 \qquad 1 < 2xy < 2$

$\Rightarrow \quad 1 < x^2 - y^2 < 2$ and $\frac{1}{2} < xy < 1$

Since we need a domain we may select

$D = \{(x,y) \mid 1 < x^2 - y^2 < 2,\ \frac{1}{2} < xy < 1,\ x \geq 0,\ y \geq 0\}$



4. a) We claim that $\lim_{z \to i} 3z - i = 2i$

for any $\varepsilon > 0$ we pick $\delta = \varepsilon/3$
so that whenever $|z - i| < \delta$

$$\begin{aligned} |3z - i - 2i| &= |3z - 3i| \\ &= 3|z - i| \\ &< 3\delta \\ &= \varepsilon \end{aligned}$$

b) We claim that $\lim_{z \to 1} (2z + \bar{z}) = 3$

for every $\varepsilon > 0$ pick $\delta = \varepsilon/3$
so that whenever $|z - 1| < \delta$

$$\begin{aligned} |2z + \bar{z} - 3| &= |2z - 2 + \bar{z} - 1| \\ &\leq 2|z - 1| + |\bar{z} - 1| \\ &= 3|z - 1| \\ &< 3\delta \\ &= \varepsilon \end{aligned}$$

c) We claim that $\lim_{z \to 0} \frac{\bar{z}^3}{z^2} = 0$

for every $\varepsilon > 0$ pick $\delta = \varepsilon$

so that whenever $|z| = |z - 0| < \delta$,

$$\left| \frac{\bar{z}^3}{z^2} - 0 \right| = \frac{|\bar{z}|^3}{|z|^2}$$

$$= \frac{|z|^3}{|z|^2}$$

$$= |z|$$

$$< \delta$$

$$= \varepsilon$$