Problem. 2.3

Proof. Prove by contradiction. Suppose that $\sqrt{2} + \sqrt{2} = p/q$ for $p, q \in \mathbb{Z}$. Then $2 + \sqrt{2} = (p/q)^2$. That means $\sqrt{2} = (p/q)^2 - 2 \in \mathbb{Q}$. That contradicts to the fact that $\sqrt{2}$ is irrational. □

Problem. 2.8

Proof. First observe that the coefficients of the polynomial on the left hand are integers. So the Rational Zeros Theorem tells us that the rational root $p/q$ which has no common factors satisfies that $p \mid 1$ and $q \mid 1$. So the only options are $p/q = \pm 1$. Plug in 1 the equality does not hold. Plug in $-1$ we get 0. So we get $-1$ the only rational root. □

Problem. 4.6(b)

Proof. Denote $\inf S = \sup S = a \in \mathbb{R}$ (because $S$ is bounded, inf, sup both exist). Note that $S \neq \emptyset$. For any $b \in S$, $a < b$ or $a > b$ or $a = b$. If $a < b$, then $\sup S = a < b \leq \sup S$, contradiction. If $a > b$, then $\inf S = a > b \leq \inf S$, contradiction. Thus $b$ must equal to $a$ for any $b$ in $S$. So the only possibility is $S = \{a\}$ ($a \in S$ because $S$ is not empty). □