

Final Exam Practice Problems

The final exam will cover: Section 7.8, 7.9, 8.2, 8.3, 9.4, 9.5, 9.7. Please be aware that you might need knowledge from earlier lectures to solve problems from these sections. The following topics are particularly important:

Convolution and its Laplace transform;
The Dirac delta and its Laplace transform;
Power series solutions to differential equations;
Homogeneous linear system;
Nonhomogeneous linear system;

BTW: If you haven't done so yet, we strongly encourage you to fill out the CAPE evaluation for our course (for both instructor and TAs). Thanks!

1. Solve $y(t)$ from the following integral equation.

$$y(t) = 1 + t - \int_0^t (t-x)y(x)dx.$$

2. Find the solution of the given initial value problem:

$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 1, \quad y'(0) = 0.$$

3. Solve the following initial value problem using a power series centered at 0. Write out the first four nonzero terms of the power series. Then find $y^{(3)}(0)$.

$$y'' - xy' - y = 0, \quad y(0) = 2, \quad y'(0) = -1.$$

4. Find the general solution of the homogeneous system:

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{pmatrix} \mathbf{x}$$

5. Use the method of undetermined coefficients to find the general solution of the nonhomogeneous system:

$$\mathbf{x}' = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

6. Use the method of variation of parameters to find the general solution of the nonhomogeneous system:

$$\mathbf{x}' = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$